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PRESTO II

A DIGITAL COMPUTER PROGRAM FOR TRAJECTORY OPTIMIZATION

by Richard C. Rosenbaum and Zoe Taulbee

Prepared by
LOCKHEED MISSILES & SPACE COMPANY
Sunnyvale, Calif.
for Langley Research Center

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FOREWORD

This report was prepared by the Aerospace Sciences Laboratory of the Lockheed Missiles and Space Company, Sunnyvale, California. It presents the documentation for a digital computer program known as PRESTO II. This program was developed by Lockheed for the Langley Research Center under NASA Contract NAS 1-5255. A FORTRAN source deck listing included with the master copy of this report completes the program documentation. PRESTO II makes use of computer subroutines that were developed for PRESTO (Program for Rapid Earth-to-Space Trajectory Optimization) under NASA Contract NAS 1-2678.* This report does not include any information on those subroutines that were described in the PRESTO final report. R. C. Rosenbaum was responsible for the work reported here. Computer programming was performed by Miss Zoe Taulbee.

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*Results of the program carried out under Contract No. NAS 1-2678 have been published as NASA CR-158 entitled "Program for Rapid Earth-to-Space Trajectory Optimization" by Robert E. Willwerth, Jr., Richard C. Rosenbaum, and Wong Chuck.

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ABSTRACT

A high-speed computer program for trajectory optimization has been developed. Program speed is obtained through the use of an approximate analytic solution to the three-dimensional equations of motion on a "cylindrical" Earth. The thrust attitude control history is assumed to follow the "linear-tangent" law. The method of steepest descent is used to determine the control parameters that maximize payload while simultaneously satisfying terminal constraints.

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1. INTRODUCTION

The development of the high-speed digital computer has made it possible to apply sophisticated numerical techniques to the problem of trajectory optimization. The steepest-descent method, in particular, has been used quite successfully to evaluate the performance of rocket boosters. For preliminary design studies, however, where a great many configurations must be evaluated, conventional numerical methods may require an excessive amount of computer time. Most of this time is taken up by repeated integration of numerous differential equations.

In PRESTO II, most of the numerical integration is eliminated through the use of an approximate analytical solution to the equations of motion. The pitch and yaw components of the thrust attitude history for the upper stages are made up of one or two segments which follow the linear-tangent rule, i.e., the tangent of the attitude angle is equal to a + bt. The first stage is assumed to fly at zero angle of attack. An iterative procedure, employing the steepest-descent method, is used to find the values of the controls which meet terminal constraints and maximize payload. In addition to the thrust attitude history, the controls that can be optimized are the initial flight path angle and azimuth, the launch time, the length of a coast between two powered stages, and the cutoff and ignition times for a restartable final stage.

It is known that the linear-tangent law provides an optimal solution to the ascent problem if one makes a "flat-Earth" assumption. Furthermore, a closed-form solution to the equations of motion is available for the "flat-Earth" case. In PRESTO II, the Earth is represented by a cylinder. The equations of motion in the pitch plane include the centrifugal and Coriolis accelerations. Lateral motion is assumed to be uncoupled from the pitch-plane motion. The linear-tangent law is applied to a thrust attitude angle that is measured with respect to the local horizontal rather than a fixed direction.

The trajectory of the first stage of the booster vehicle is integrated numerically. The first stage trajectory is a function only of the initial flight path angle and launch weight because of the zero angle of attack assumption. A table of trajectory variables at the end of the first stage as a function of flight path angle is formed. This table is then used to represent the first stage during the optimization process.

After an optimum trajectory has been obtained on PRESTO II, the optimum controls are used to run a numerically integrated trajectory on PRESTO. Terminal conditions will, in general, not be satisfied because of the approximations in the closed-form solution. A backward guidance run and a final guided run are then made on PRESTO to produce a numerically integrated trajectory which meets terminal conditions and provides a near-optimum payload.

A description of the program, together with flow diagrams for the larger subroutines, is presented in Section 2. The nomenclature introduced in the PRESTO II subroutines is defined in Section 3. Section 4 serves as a manual for operating the program. The optimization equations are derived in Section 5 and Section 6 provides a derivation of the approximate solution to the equations of motion.

PRESTO II makes use of the nomenclature and logical arrangement of PRESTO to as great a degree as possible. It is therefore suggested that the reader have the PRESTO final report available for reference.

2. PROGRAM DESCRIPTION

PRESTO II is written in the Fortran IV language. The program is composed of two distinct parts. One part is the DELTA version of the original PRESTO program with a few subroutines altered. The other part is composed of the new subroutines associated with PRESTO II. In this section, the two parts of the program will be referred to as PRESTO and PRESTO II.

PRESTO II is comprised of the following subroutines:

MAIN	CONVRT
STAGE 1	VALUAT
PERTRB	LOOK 2
TRAJ 2	OPCNL 2
NSTOP 2	RUNGEK
COAST 2	NTEGRT
CONC	LIFT
MEQ 2	EQUATN

The following subroutines are taken from the PRESTO program:

PRESTO (MAIN of PRESTO)	MISCON *
TRAJ	SYMVRT *
PCAL	LOOK *
OPCNL	INER *
MEQ	ATMOS *
INSTOP	LUNEPH *
ICS	PLANEP *
DEQ	REIN *
COAST	OPTION *
RKAD	WIC *

The function of each of the PRESTO II subroutines is discussed here.

Subroutine Functions

MAIN

The operations performed in the PRESTO II MAIN program are similar to those performed in the PRESTO MAIN program. Additional calculations are made to set up the initial values of the control and vehicle parameters. The closed-form lift-off calculation has been removed to a separate subroutine. At the end of the PRESTO II iterations, subroutine PRESTO is called to begin the numerical integration of the trajectory.

STAGE 1

The trajectory for the first stage of a vehicle launched from the Earth's surface is computed here. A table of trajectory variables at the end of the first stage, as a function of the initial flight path angle, is stored in the two dimensional array, GTABLE. At the beginning of each case, two entries are made in GTABLE. The first is for the initial flight path angle, γ_I , and the second is for γ_I plus DELGAM. The slope of the trajectory variables is then computed and stored in QGTABL. Thereafter, new entries in GTABLE and QGTABL are made only when the initial flight path angle falls outside of the region already included in the table. There is room for twenty-five entries on each side of the initial table entry.

If the fixed-final stage option is being used, so that the launch weight is adjustable, the influence of launch weight changes must also be determined. Initially, this is done by perturbing the launch mass by .001 times the initial mass. The change in the first stage burnout conditions due to this mass change is stored in the OKMS array. There is an entry in OKMS for every entry in GTABLE. Thus, every initial flight path angle has a mass slope associated with it. Furthermore, for every table entry, the launch mass is stored in the array GMASS. When a table lookup is made, the current launch mass is compared with the GMASS associated with the current flight path angle. This mass difference is multiplied by the current value of the mass slope, OKM 1, to obtain the correction in first stage burnout conditions due to the change in launch weight. A new mass slope is computed after NEWKM 1 entries have been made in GTABLE. NEWKM 1 is input by the user.

The same sort of correction is made if the launch azimuth is adjustable. The current value of the azimuth slope is stored in the second half of OKM 1. A new slope is computed whenever the initial azimuth changes by more than DELPSI. DELPSI is also input by the user.

The trajectory variables at the end of the first stage are placed in F1 and are returned to the calling program by means of an argument list.

A flow chart for STAGE 1 is at the end of this section.

PERTRB

The sensitivities of the terminal constraints and stopping parameters to changes in the control variables are computed here. PERTRB replaces the backward integration performed in PRESTO. Each of the controls is perturbed by an amount, DELCON, set by the user. A trajectory is then computed. The perturbed terminal conditions are compared with the nominal terminal conditions to determine the influence of the control change.

A flow chart for PERTRB is at the end of this section.

TRAJ 2

TRAJ 2 performs the same operations as does TRAJ in PRESTO. TRAJ 2 is considerably smaller than TRAJ because of the absence of a backward trajectory. The subroutine has also been shortened through the use of a common area for determining the trajectory during each stage.

A flow chart for TRAJ 2 is at the end of this section.

NSTOP 2

This subroutine interpolates to the correct stopping parameter.

COAST 2

Motion across a coast stage is determined here. The equations used are from the COAST routine in PRESTO.

CONC

The control variables to be used on the next trajectory are computed.

MEQ 2

The matrices required to compute the control changes in CONC are computed. See Section 5 for a derivation of the equations used.

CONVRT

The trajectory variables associated with the cylindrical coordinate system are converted to the geographic coordinates used in PRESTO. The equations used are given in Section 6.

VALUAT

The derivatives used in the numerical integration of the first stage are computed.

LOOK 2

This routine is similar to LOOK. It provides the linear interpolation used in STAGE 1.

OPCNL 2

The trajectory variables at the beginning and end of each stage are output.

RUNGEK

This subroutine provides the numerical integration formulas used for the first stage. It is similar to RKAD.

NTEGRT

The integration of the first stage is controlled.

LIFT

The closed-form lift-off calculation is performed.

EQUATN

The approximate analytic solution to the equations of motion is computed.

The results are stored in the array, UVH, and are returned to the calling program through an argument list.

The following PRESTO subroutines have been changed.

PRESTO

PRESTO is the subroutine name for the MAIN routine of the PRESTO program. The initial setup operations now appear in the PRESTO II MAIN and have been removed from subroutine PRESTO. When the program is under the control of PRESTO, it runs a nominal trajectory, a backward guidance trajectory, and a final guided run. Control is then returned to the MAIN program.

TRAJ

A change has been made so that an intermediate constraint (circular park orbit or radius of perigee) will be applied on the final guided run if it was used in PRESTO II.

PCAL

The thrust attitude angles are computed from the PRESTO II angles on the nominal trajectory. This computation replaces the table look-up in PRESTO. The equations used here are derived in Section 6.

REIN and WIC

Changes associated with the PRESTO II data requirements have been made.

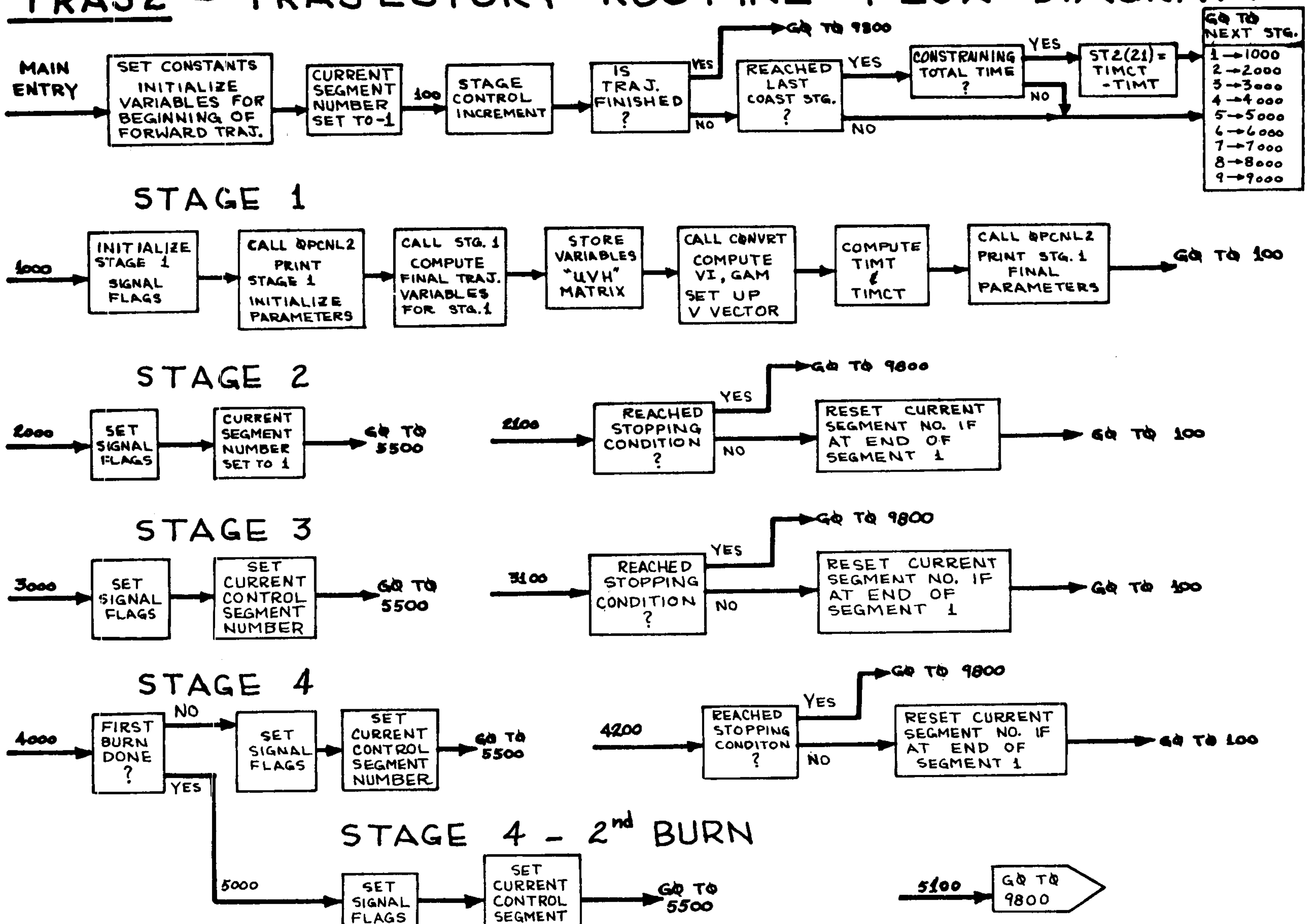
Use of PRESTO II Without PRESTO

If a numerically integrated trajectory is not required, the size of the program can be greatly reduced by eliminating many of the PRESTO subroutines. PRESTO II can run by itself using all of the PRESTO II subroutines plus those PRESTO subroutines identified by an asterisk on page 2-1. A dummy subroutine PRESTO is also required.

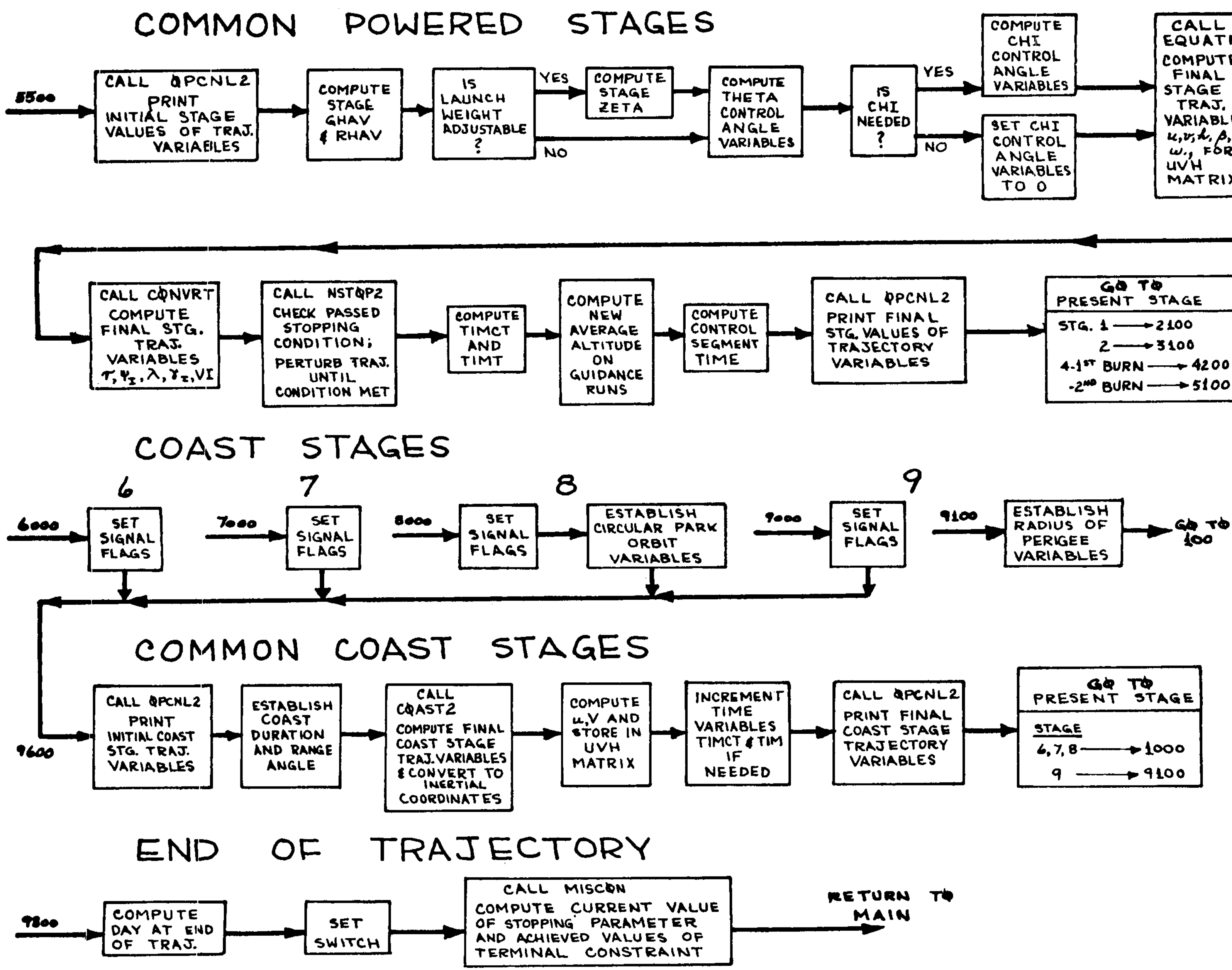
Use of COMMON Variables

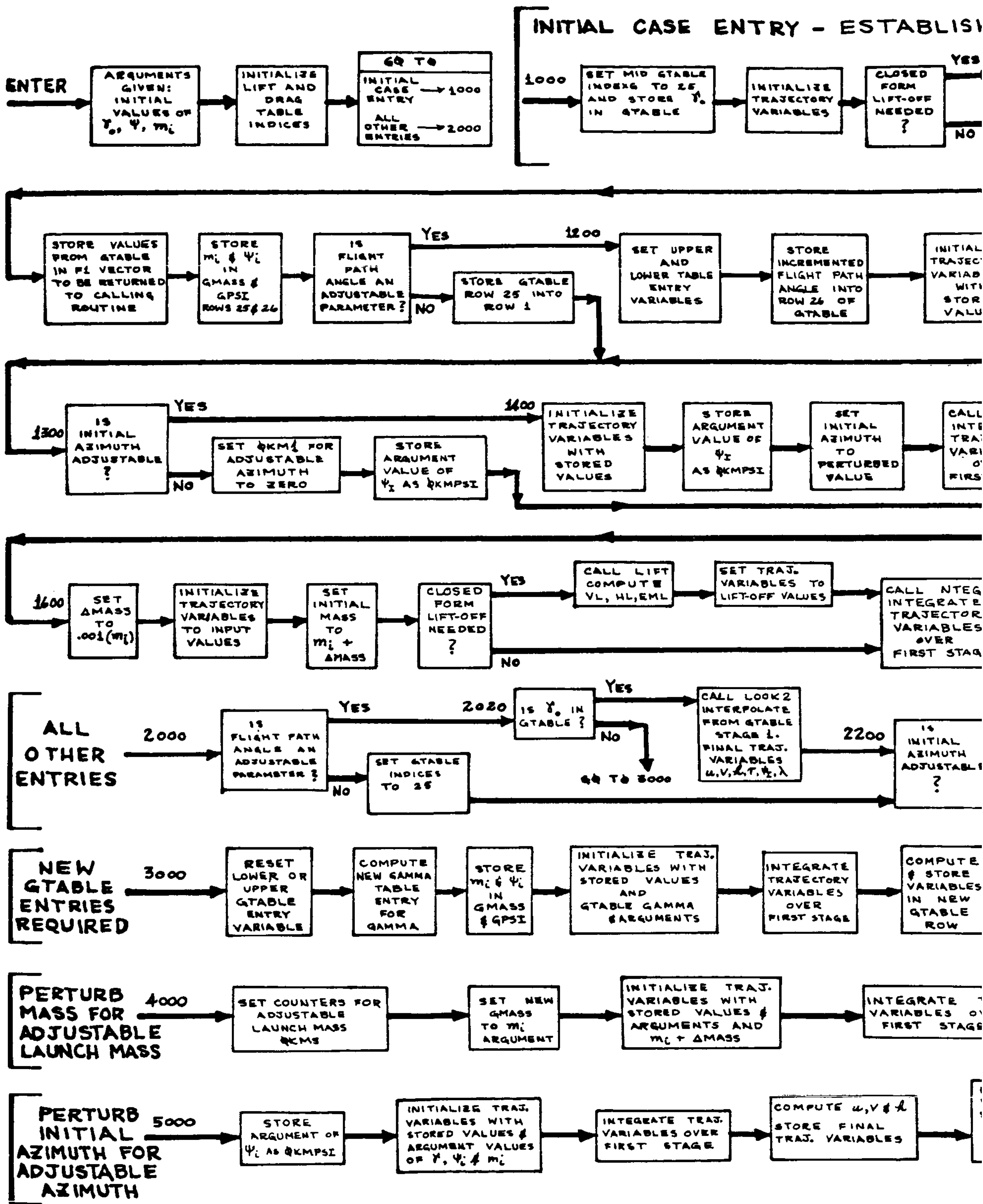
The variables stored in COMMON have been separated into three groups. One group, known as the common COMMON, is used in all subroutines that employ COMMON. A second group of variables which are used only in PRESTO II subroutines appears at the end of common COMMON in the PRESTO II subroutines. A third group of variables which are used only in the PRESTO subroutines appears at the end of common COMMON in the PRESTO subroutines.

TRAJ2 - TRAJECTORY ROUTINE FLOW DIAGRAM

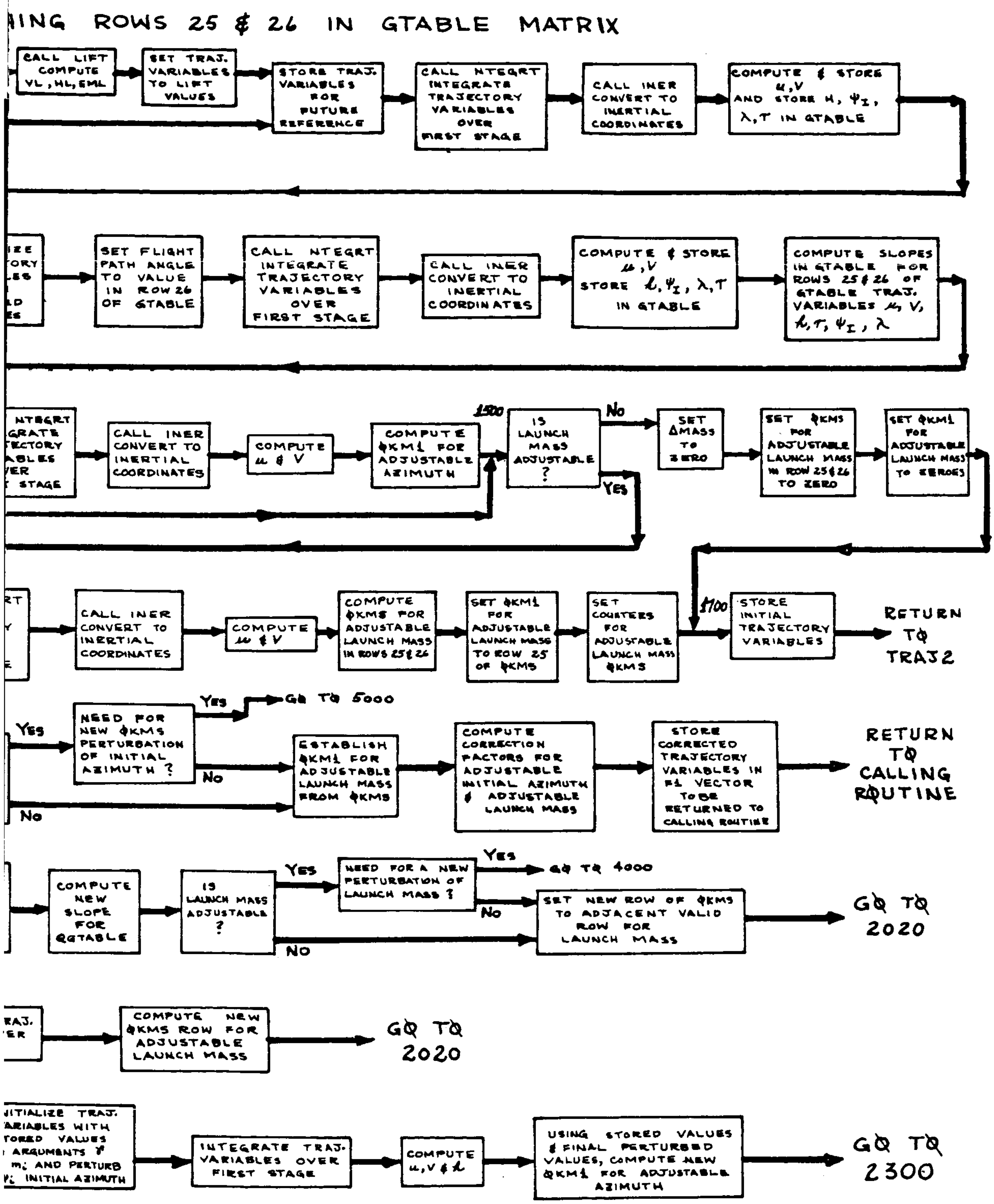


TRAJ2

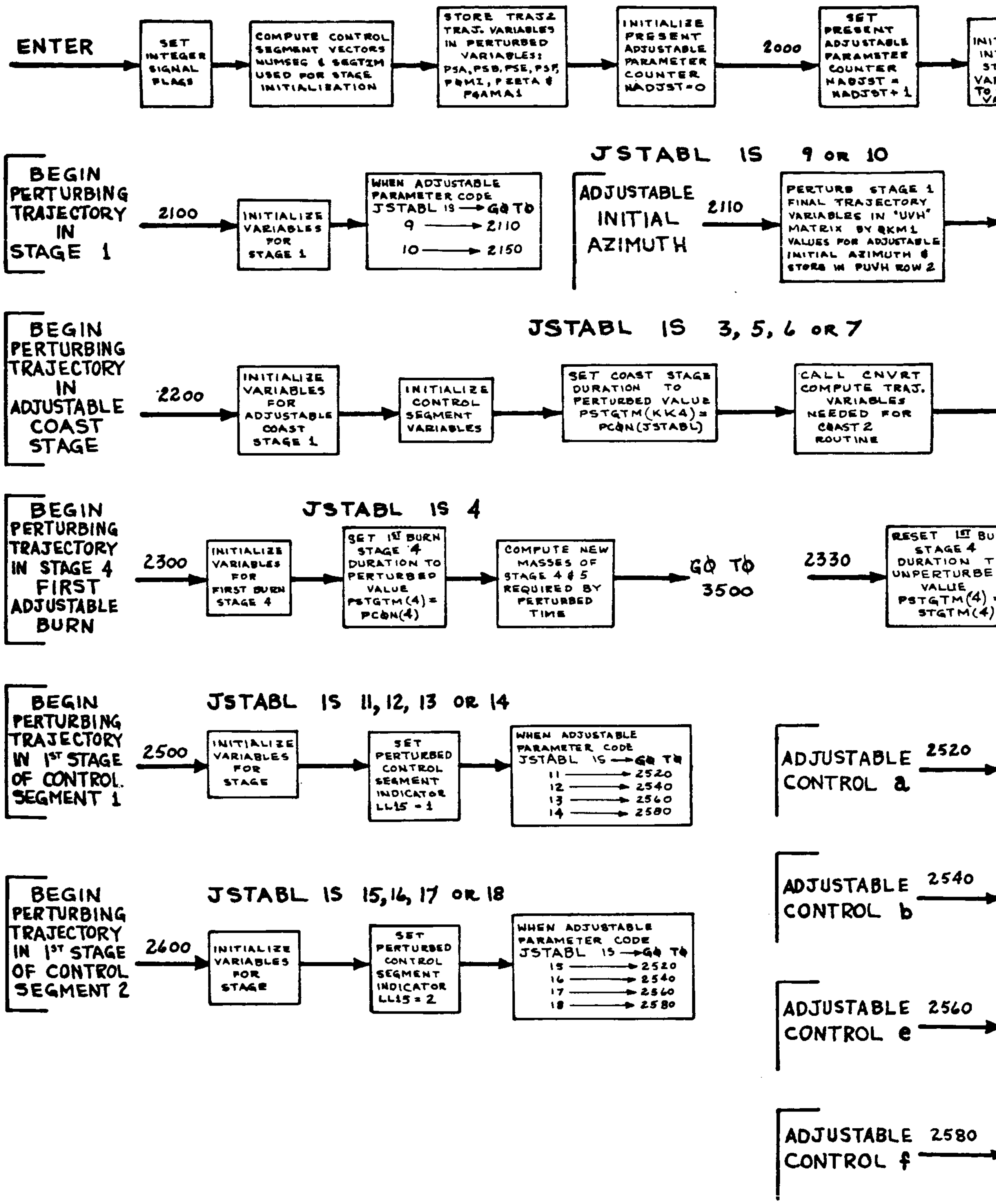

COMMON POWERED STAGES
5500
CALL ØPCNL2 PRINT INITIAL STAGE VALUES OF TRAJ. VARIABLES
COMPUTE STAGE GHAV & RHAV
IS LAUNCH WEIGHT ADJUSTABLE ?
YES
NO
COMPUTE STAGE ZETA
COMPUTE THETA CONTROL ANGLE VARIABLES
IS CHI NEEDED ?
YES
NO
COMPUTE CHI CONTROL ANGLE VARIABLES
SET CHI CONTROL ANGLE VARIABLES TO 0
CALL EQUATN COMPUTE FINAL STAGE TRAJ. VARIABLES u, v, h, β, Λ, ω, FOR UVH MATRIX
CALL CØNVRT COMPUTE FINAL STG. TRAJ. VARIABLES τ, ψ, λ, γ, VI
CALL NSTØP2 CHECK PASSED STOPPING CONDITION; PERTURB TRAJ. UNTIL CONDITION MET
COMPUTE TIMCT AND TIMT
COMPUTE NEW AVERAGE ALTITUDE ON GUIDANCE RUNS
COMPUTE CONTROL SEGMENT TIME
CALL ØPCNL2 PRINT FINAL STG. VALUES OF TRAJECTORY VARIABLES
GØ TØ PRESENT STAGE
STG. 1 → 2100
2 → 3100
4-1ST BURN → 4200
-2ND BURN → 5100
COAST STAGES
6
7
8
9
6000
SET SIGNAL FLAGS
7000
SET SIGNAL FLAGS
8000
SET SIGNAL FLAGS
ESTABLISH CIRCULAR PARK ORBIT VARIABLES
9000
SET SIGNAL FLAGS
9100
ESTABLISH RADIUS OF PERIGEE VARIABLES
GØ TØ 100
COMMON COAST STAGES
9600
CALL ØPCNL2 PRINT INITIAL COAST STG. TRAJ. VARIABLES
ESTABLISH COAST DURATION AND RANGE ANGLE
CALL CØAST2 COMPUTE FINAL COAST STAGE TRAJ. VARIABLES & CONVERT TO INERTIAL COORDINATES
COMPUTE u, V AND STORE IN UVH MATRIX
INCREMENT TIME VARIABLES TIMCT & TIM IF NEEDED
CALL ØPCNL2 PRINT FINAL COAST STAGE TRAJECTORY VARIABLES
GØ TØ PRESENT STAGE
STAGE
6, 7, 8 → 1000
9 → 9100
END OF TRAJECTORY
9800
COMPUTE DAY AT END OF TRAJ.
SET SWITCH
CALL MISCØN COMPUTE CURRENT VALUE OF STOPPING PARAMETER AND ACHIEVED VALUES OF TERMINAL CONSTRAINT
RETURN TØ MAIN

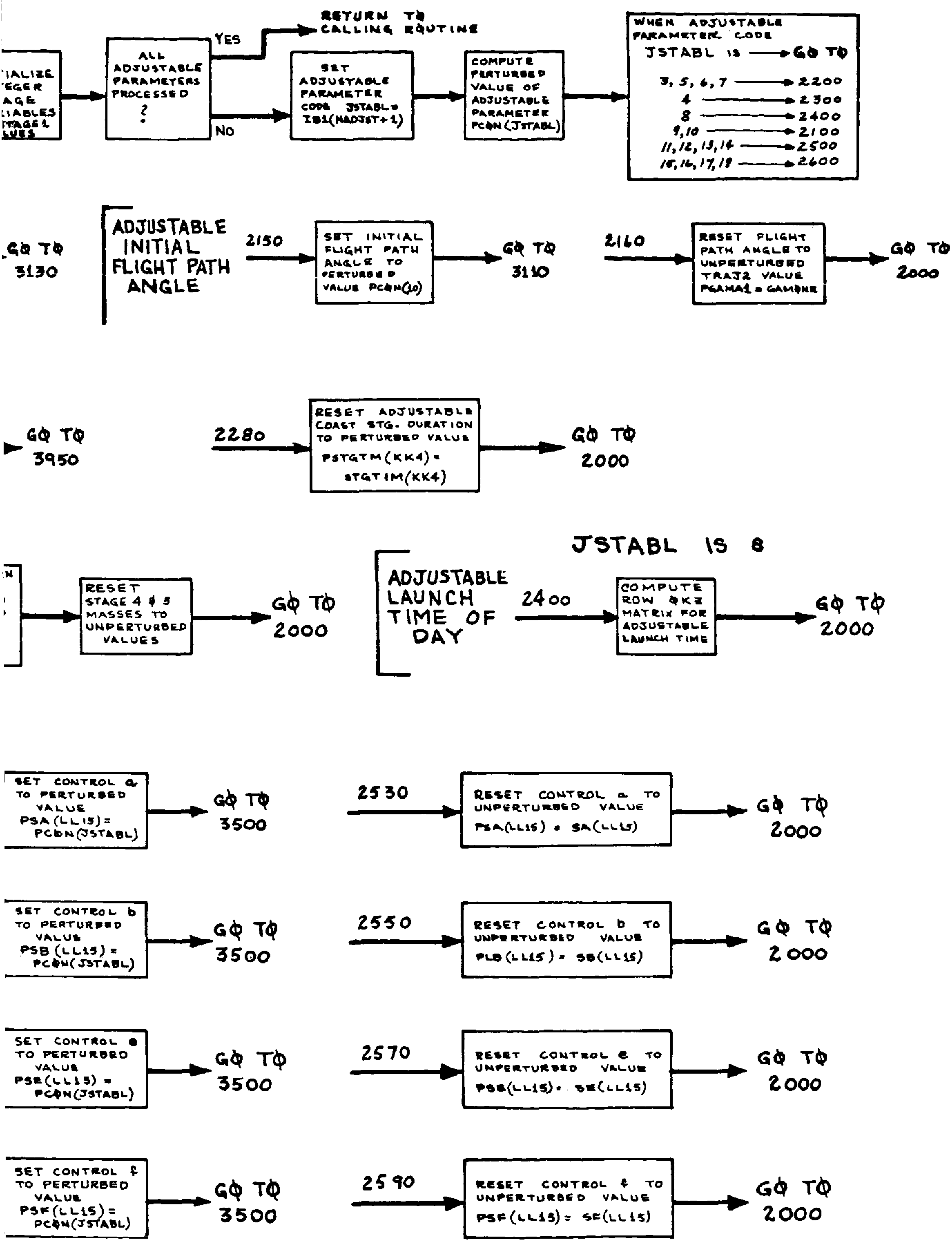


INITIAL CASE ENTRY - ESTABLIS
ENTER
ARGUMENTS GIVEN: INITIAL VALUES OF γ_o, Ψ, m_i
INITIALIZE LIFT AND DRAG TABLE INDICES
GO TO
INITIAL CASE ENTRY → 1000
ALL OTHER ENTRIES → 2000
1000
SET MID GTABLE INDEXG TO 25 AND STORE γ_o IN GTABLE
INITIALIZE TRAJECTORY VARIABLES
CLOSED FORM LIFT-OFF NEEDED ?
YES
NO
STORE VALUES FROM GTABLE IN FI VECTOR TO BE RETURNED TO CALLING ROUTINE
STORE m_i & Ψ_i IN GMASS & GPSI ROWS 25 & 26
IS FLIGHT PATH ANGLE AN ADJUSTABLE PARAMETER ?
YES
NO
1200
STORE GTABLE ROW 25 INTO ROW 1
SET UPPER AND LOWER TABLE ENTRY VARIABLES
STORE INCREMENTED FLIGHT PATH ANGLE INTO ROW 26 OF GTABLE
INITIAL TRAJEC VARIAB WIT STOR VALU
1300
IS INITIAL AZIMUTH ADJUSTABLE ?
YES
NO
1400
SET ΦKM1 FOR ADJUSTABLE AZIMUTH TO ZERO
STORE ARGUMENT VALUE OF Ψ_I AS ΦKMPSI
INITIALIZE TRAJECTORY VARIABLES WITH STORED VALUES
STORE ARGUMENT VALUE OF Ψ_I AS ΦKMPSI
SET INITIAL AZIMUTH TO PERTURBED VALUE
CALL INTE TRA VARI O FIRS
1600
SET ΔMASS TO .001(m_i)
INITIALIZE TRAJECTORY VARIABLES TO INPUT VALUES
SET INITIAL MASS TO m_i + ΔMASS
CLOSED FORM LIFT-OFF NEEDED ?
YES
NO
CALL LIFT COMPUTE VL, HL, EML
SET TRAJ. VARIABLES TO LIFT-OFF VALUES
CALL NTEG INTEGRATE TRAJECTOR VARIABLES OVER FIRST STAG
ALL OTHER ENTRIES
2000
IS FLIGHT PATH ANGLE AN ADJUSTABLE PARAMETER ?
YES
NO
2020
IS γ_o IN GTABLE ?
YES
NO
SET GTABLE INDICES TO 25
GO TO 3000
CALL LOOK2 INTERPOLATE FROM GTABLE STAGE 1. FINAL TRAJ. VARIABLES u, V, h, T, Ψ, λ
2200
IS INITIAL AZIMUTH ADJUSTABLE ?
NEW GTABLE ENTRIES REQUIRED
3000
RESET LOWER OR UPPER GTABLE ENTRY VARIABLE
COMPUTE NEW GAMMA TABLE ENTRY FOR GAMMA
STORE m_i & Ψ_i IN GMASS & GPSI
INITIALIZE TRAJ. VARIABLES WITH STORED VALUES AND GTABLE GAMMA & ARGUMENTS
INTEGRATE TRAJECTORY VARIABLES OVER FIRST STAGE
COMPUTE & STORE VARIABLES IN NEW GTABLE ROW
PERTURB MASS FOR ADJUSTABLE LAUNCH MASS
4000
SET COUNTERS FOR ADJUSTABLE LAUNCH MASS ΦKMS
SET NEW GMASS TO m_i ARGUMENT
INITIALIZE TRAJ. VARIABLES WITH STORED VALUES & ARGUMENTS AND m_i + ΔMASS
INTEGRATE T VARIABLES O FIRST STAGE
PERTURB INITIAL AZIMUTH FOR ADJUSTABLE AZIMUTH
5000
STORE ARGUMENT OF Ψ_i AS ΦKMPSI
INITIALIZE TRAJ. VARIABLES WITH STORED VALUES & ARGUMENT VALUES OF γ, Ψ_i & m_i
INTEGRATE TRAJ. VARIABLES OVER FIRST STAGE
COMPUTE u, V & h STORE FINAL TRAJ. VARIABLES

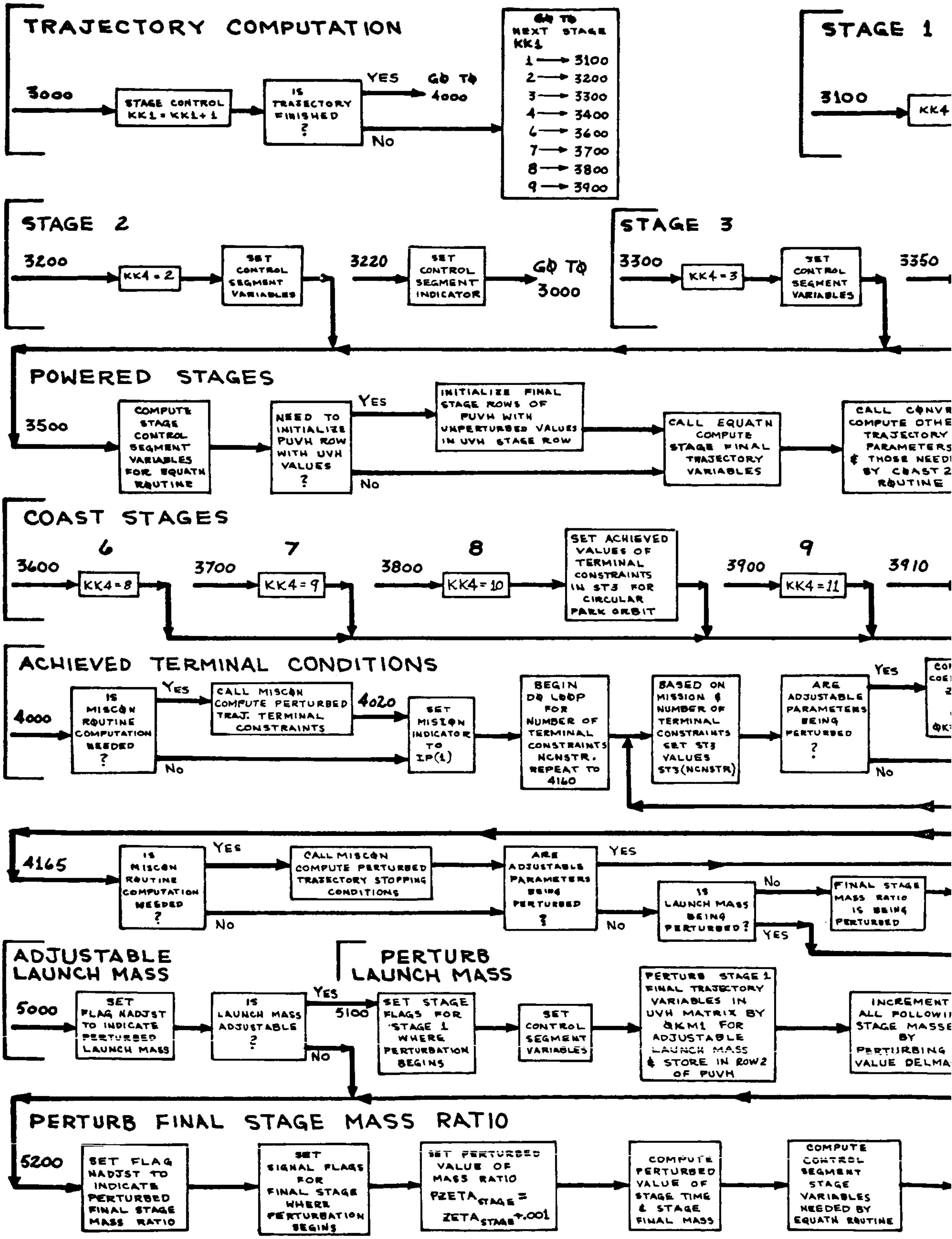


SUBROUTINE STAGE 1

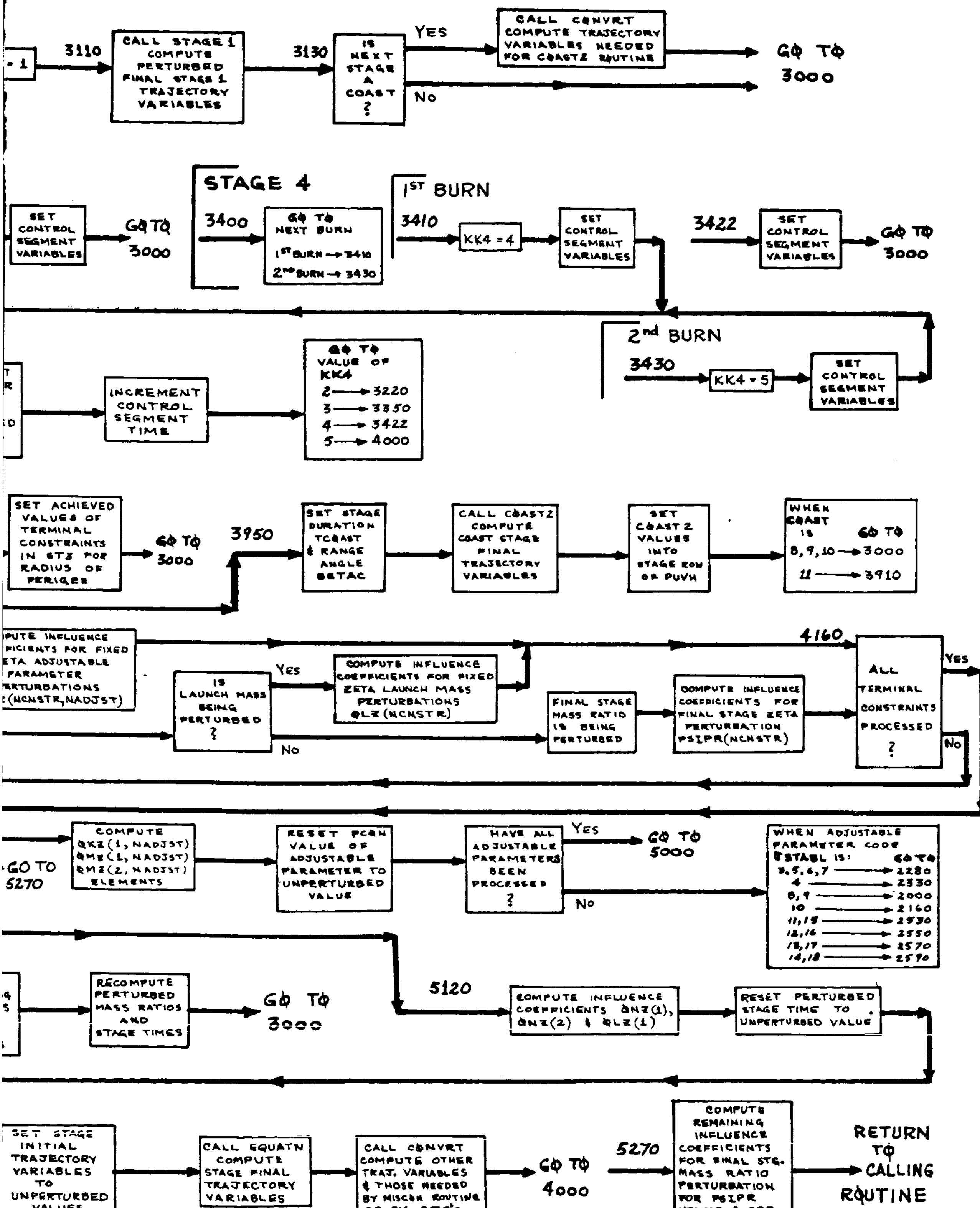

ENTER
SET INTEGER SIGNAL FLAGS
COMPUTE CONTROL SEGMENT VECTORS NUMSEG & SEGTIM USED FOR STAGE INITIALIZATION
STORE TRAJ2 TRAJ. VARIABLES IN PERTURBED VARIABLES: PSA, PSB, PSE, PSF, PRMI, PZETA & PGAMA1
INITIALIZE PRESENT ADJUSTABLE PARAMETER COUNTER NADJST=0
2000
SET PRESENT ADJUSTABLE PARAMETER COUNTER NADJST = NADJST + 1
BEGIN PERTURBING TRAJECTORY IN STAGE 1
2100
INITIALIZE VARIABLES FOR STAGE 1
WHEN ADJUSTABLE PARAMETER CODE JSTABL IS → GO TO
9 → 2110
10 → 2150
JSTABL IS 9 OR 10
ADJUSTABLE INITIAL AZIMUTH
2110
PERTURB STAGE 1 FINAL TRAJECTORY VARIABLES IN 'UVH' MATRIX BY QKM1 VALUES FOR ADJUSTABLE INITIAL AZIMUTH & STORE IN PUVH ROW 2
BEGIN PERTURBING TRAJECTORY IN ADJUSTABLE COAST STAGE
2200
JSTABL IS 3, 5, 6 OR 7
INITIALIZE VARIABLES FOR ADJUSTABLE COAST STAGE 1
INITIALIZE CONTROL SEGMENT VARIABLES
SET COAST STAGE DURATION TO PERTURBED VALUE PSTGTM(KK4) = PCON(JSTABL)
CALL CNVRT COMPUTE TRAJ. VARIABLES NEEDED FOR COAST 2 ROUTINE
BEGIN PERTURBING TRAJECTORY IN STAGE 4 FIRST ADJUSTABLE BURN
JSTABL IS 4
2300
INITIALIZE VARIABLES FOR FIRST BURN STAGE 4
SET 1ST BURN STAGE 4 DURATION TO PERTURBED VALUE PSTGTM(4) = PCON(4)
COMPUTE NEW MASSES OF STAGE 4 & 5 REQUIRED BY PERTURBED TIME
GO TO 3500
2330
RESET 1ST BU STAGE 4 DURATION T UNPERTURBE VALUE PSTGTM(4) STGTM(4)
BEGIN PERTURBING TRAJECTORY IN 1ST STAGE OF CONTROL SEGMENT 1
JSTABL IS 11, 12, 13 OR 14
2500
INITIALIZE VARIABLES FOR STAGE
SET PERTURBED CONTROL SEGMENT INDICATOR LL15 = 1
WHEN ADJUSTABLE PARAMETER CODE JSTABL IS → GO TO
11 → 2520
12 → 2540
13 → 2560
14 → 2580
ADJUSTABLE CONTROL a
2520
BEGIN PERTURBING TRAJECTORY IN 1ST STAGE OF CONTROL SEGMENT 2
JSTABL IS 15, 16, 17 OR 18
2600
INITIALIZE VARIABLES FOR STAGE
SET PERTURBED CONTROL SEGMENT INDICATOR LL15 = 2
WHEN ADJUSTABLE PARAMETER CODE JSTABL IS → GO TO
15 → 2520
16 → 2540
17 → 2560
18 → 2580
ADJUSTABLE CONTROL b
2540
ADJUSTABLE CONTROL e
2560
ADJUSTABLE CONTROL f
2580



SUBROUTINE PERTRB


TRAJECTORY COMPUTATION
3000
STAGE CONTROL KK1 = KK1 + 1
IS TRAJECTORY FINISHED ?
YES
GO TO 4000
NO
GO TO NEXT STAGE KK1
1 → 3100
2 → 3200
3 → 3300
4 → 3400
6 → 3600
7 → 3700
8 → 3800
9 → 3900
STAGE 1
3100
KK4
STAGE 2
3200
KK4 = 2
SET CONTROL SEGMENT VARIABLES
3220
SET CONTROL SEGMENT INDICATOR
GO TO 3000
STAGE 3
3300
KK4 = 3
SET CONTROL SEGMENT VARIABLES
3350
POWERED STAGES
3500
COMPUTE STAGE CONTROL SEGMENT VARIABLES FOR EQUATN ROUTINE
NEED TO INITIALIZE PUVH ROW WITH UVH VALUES ?
YES
NO
INITIALIZE FINAL STAGE ROWS OF PUVH WITH UNPERTURBED VALUES IN UVH STAGE ROW
CALL EQUATN COMPUTE STAGE FINAL TRAJECTORY VARIABLES
CALL CONVR COMPUTE OTHE TRAJECTORY PARAMETERS & THOSE NEEDI BY COAST 2 ROUTINE
COAST STAGES
6
3600
KK4 = 8
7
3700
KK4 = 9
8
3800
KK4 = 10
SET ACHIEVED VALUES OF TERMINAL CONSTRAINTS IN ST3 FOR CIRCULAR PARK ORBIT
9
3900
KK4 = 11
3910
ACHIEVED TERMINAL CONDITIONS
4000
IS MISCON ROUTINE COMPUTATION NEEDED ?
YES
NO
CALL MISCON COMPUTE PERTURBED TRAJ. TERMINAL CONSTRAINTS
4020
SET MISION INDICATOR TO IP(1)
BEGIN DO LOOP FOR NUMBER OF TERMINAL CONSTRAINTS NCNSTR. REPEAT TO 4160
BASED ON MISSION & NUMBER OF TERMINAL CONSTRAINTS SET ST3 VALUES ST3(NCNSTR)
ARE ADJUSTABLE PARAMETERS BEING PERTURBED ?
YES
NO
4165
IS MISCON ROUTINE COMPUTATION NEEDED ?
YES
NO
CALL MISCON COMPUTE PERTURBED TRAJECTORY STOPPING CONDITIONS
ARE ADJUSTABLE PARAMETERS BEING PERTURBED ?
YES
NO
IS LAUNCH MASS BEING PERTURBED ?
NO
YES
FINAL STAGE MASS RATIO IS BEING PERTURBED
ADJUSTABLE LAUNCH MASS
5000
SET FLAG NADJST TO INDICATE PERTURBED LAUNCH MASS
IS LAUNCH MASS ADJUSTABLE ?
YES
NO
PERTURB LAUNCH MASS
5100
SET STAGE FLAGS FOR STAGE 1 WHERE PERTURBATION BEGINS
SET CONTROL SEGMENT VARIABLES
PERTURB STAGE 1 FINAL TRAJECTORY VARIABLES IN UVH MATRIX BY ΔKM1 FOR ADJUSTABLE LAUNCH MASS & STORE IN ROW 2 OF PUVH
INCREMENT ALL FOLLOWI STAGE MASSE BY PERTURBING VALUE DELMA
PERTURB FINAL STAGE MASS RATIO
5200
SET FLAG NADJST TO INDICATE PERTURBED FINAL STAGE MASS RATIO
SET SIGNAL FLAGS FOR FINAL STAGE WHERE PERTURBATION BEGINS
SET PERTURBED VALUE OF MASS RATIO PZETA STAGE = ZETA STAGE +.001
COMPUTE PERTURBED VALUE OF STAGE TIME & STAGE FINAL MASS
COMPUTE CONTROL SEGMENT STAGE VARIABLES NEEDED BY EQUATN ROUTINE

PERTRB (CONT'D.)



3. DEFINITIONS OF PRESTO II VARIABLES

The variables introduced in the PRESTO II subroutines are defined here.

CA Pitch plane steady-state control angle for current stage

CB Pitch plane slope for current stage

CE Yaw plane steady-state control angle for current stage

CF Yaw plane slope for current stage

CON Vector of current values of adjustable parameters

3. Length of coast after first burn in stage 4
4. Length of first burn in stage 4
5. Length of coast after stage 3
6. Length of coast after stage 2
7. Length of coast after stage 1
8. Launch time of day
9. Initial azimuth
10. Initial flight path angle
11. Initial value of pitch plane control variable for segment one (a_1)
12. Slope of pitch plane control variable for segment one (b_1)
13. Initial value of yaw plane control variable for segment one (e_1)
14. Slope of yaw plane control variable for segment one (f_1)
15. Initial value of pitch plane control variable for segment two (a_2)
16. Slope of pitch plane control variable for segment two (b_2)
17. Initial value of yaw plane control variable for segment two (e_2)
18. Slope of yaw plane control variable for segment two (f_2)

CNTRL 1 Data block 40

CNTRL 2 Data block 41

DELCON Vector of magnitudes of perturbations of adjustable parameters

DELGAM Data block 43

DELMAS Magnitude of launch mass perturbation

DELPSI Data block 43

DELTA Data block 42

DTR Conversion factor for converting degrees to radians

EQC Vector of powered stages exhaust velocities

EQT Vector of thrusts for powered stages

EXTRA Vector of miscellaneous variables

- 1 Stored value of ST3(1) preceding circular park orbit
- 2-7 Stored values of unperturbed stage 1 burnout conditions used to determine sensitivity of stage 1 burnout conditions to initial azimuth
- 8 Stored value of GAMI at start of the circular park orbit
- 9 Stored value of X(3) at start of the circular park orbit
- 10 Stored value of X(4) at the end of the trajectory

F1 Vector established in STAGE 1 routine with stage 1 burnout values of u, v, h, τ, ψ_I, and λ, and returned to calling routine

GAMONE Initial flight path angle

GMASS Launch mass vector associated with GTABLE row entries (for initial flight path angle γ_I) of integrated first stage final trajectory variables u, v, h, τ, ψ_I, and λ (in STAGE 1 routine)

GPSI Initial azimuth vector associated with GTABLE row entries (for initial flight path angle γ_I) of integrated first stage final trajectory variables u, v, h, τ, ψ_I, and λ (in STAGE 1 routine)

GTABLE Final trajectory variables u, v, h, τ, ψ_I, and λ for values of initial flight path angle γ_I. A table matrix established and used by STAGE 1 routine

HAV Vector of average altitudes of powered stages

HAVI Data block 45

ISEG Data block 44

J6 Used in MEQ2 to indicate whether optimization indicator shall be computed

JSTABL	Code of adjustable parameter being processed in PERTRB routine
KASE	Used in STAGE 1 routine to indicate whether initialization of GTABLE is needed
KNSTRN	Code of terminal constraint being processed in PERTRB routine
NADJST	Sequence number of adjustable parameter being processed in PERTRB routine
NCNSTR	Sequence number of terminal constraint being processed in PERTRB routine
NEWKM1	Data block 43
NUMSEG	A vector indicating for each stage its relation to control segments: 1 for being included in control segment one; 2 for being in control segment two; and 0 for not being included in a control segment
LL1	First powered stage in segment one (KK4)
LL2	Last powered stage in segment one (KK4)
LL3	First powered stage in segment two (KK4)
LL4	Current segment number
LL5	Number of segments
LL6	Last stage used in a forward trajectory (KK1)
LL7	First powered stage in segment one (KK1)
LL8	First powered stage in segment two (KK1)
LL9	Adjustable coast stage (KK1)
LL10	First burn in fourth stage (KK1)
LL11	Number of angle controls plus other adjustable parameters
LL12	MISCON calling key 0 - do not call MISCON 1 - call MISCON
LL13	Adjustable coast stage (KK2)
LL14	Last powered stage in trajectory (KK4)
LL15	Segment indicator of adjustable controls being perturbed in PERTRB
LL16	Indicates presence of stage 8 0 - stage 8 not used 1 - stage 8 used

LL17 Available

LL18 Trajectory parameter printout key
1 - initial stage 1 values
2 - final stage 1 values
3 - initial powered stages
4 - final powered stages
5 - initial coast stages
6 - final coast stages

LL19 Weight (OMI_i) subscript for printout (OPCNL2)

LL20 Nominal or perturbed trajectory
0 - nominal trajectory
1 - perturbed trajectory

LL21 Current value for the powered stage immediately following the last coast stage. If there is no coast stage, it is equal to LL7.

LL22 Assigned go to integer variable for LL21

LL23 Available

LL24 Available

LL25 Available

LL26 Available

LL27 Available

LL28 Available

LL29 Available

LL30 Available

OKM1 First six elements are sensitivities of stage 1 burnout conditions to launch mass. Other six elements are sensitivities of stage 1 burnout conditions to initial azimuth.

OKMS Stored values of first six elements of OKM1

OKS Sensitivity matrix of terminal constraints to adjustable parameters for a fixed value of the stopping parameter

OKZ Sensitivity matrix of terminal constraints to adjustable parameters for a fixed value of the final stage mass ratio

OLS	Sensitivity of terminal constraints to launch mass for a fixed value of the stopping parameter
OLZ	Sensitivity of terminal constraints to launch mass for a fixed value of the final stage mass ratio
OMI	Vector of each powered stage's initial and final masses
OMZ	Sensitivity of stopping parameter to adjustable parameters for a fixed value of the final stage mass ratio
ONZ	Sensitivity of stopping parameter to launch mass for a fixed value of the final stage mass ratio
PCON	Vector of values of achieved adjustable parameters which are perturbed by the vector DELCON in the PERTRB routine
PGAMA1	Value of initial flight path angle used by PERTRB routine for computations of perturbed trajectories
POMI	Values of initial and final masses of powered stages used by PERTRB routine for computations of perturbed trajectories
PPAY	Sensitivities relating final weight to errors in terminal constraints
PSA	Vector of pitch plane steady-state control angles for control segments one and two used by PERTRB routine for computations of perturbed trajectories
PSB	Vector of pitch plane slopes for control segments one and two used by PERTRB routine for computations of perturbed trajectories
PSE	Vector of yaw plane steady-state control angles for control segments one and two used by PERTRB routine for computations of perturbed trajectories
PSF	Vector of yaw plane slopes for control segments one and two used by PERTRB routine for computations of perturbed trajectories
PSIONE	Initial azimuth
PSIPR	Vector of derivatives of terminal constraints with respect to last stage mass ratio
PSTGTM	Vector of stage time durations for all stages used by PERTRB routine for computations of perturbed trajectories

PUVH	Matrix of trajectory variables u, v, h, τ, ψ_I, λ, β, w, and Λ for initial and final values of each stage used by PERTRB routine for computation of perturbed trajectories
PZETA	Vector of powered stages mass ratios used by PERTRB routine for computation of perturbed trajectories
QGTABL	Matrix of slopes associated with GTABLE matrix entries in STAGE 1
RHAV	Vector of average radii of powered stages
SA	Vector of pitch plane steady-state control angles for control segments one and two
SB	Vector of pitch plane sbpes for control segments one and two
SE	Vector of yaw plane steady-state control angles for control segments one and two
SEGTIM	Vector with values of control segment time duration up to that stage for each stage included in a control segment, with a zero stored in stages not included in a control segment
SF	Vector of yaw plane slopes for control segments one and two
SF1	Vector returned by STAGE 1 routine with stage one burnout values of u, v, h, τ, ψ_I and λ
SHAV	Vector of saved values of HAV vector
SOMI	Vector of saved values of OMI vector
SPR	Derivative of stopping parameter with respect to mass ratio
SST3	Vector of saved values of ST3 vector
STGTIM	Vector of stage time duration for all stages
SWFUEL	Correct fuel weight in the final stage
TABLEG	Data block 43
TCOAST	Time duration of a coast stage
TCTIM	Total time in each control segment initially
TIM	Current value of control segment time
UVH	Matrix of trajectory variables u, v, h, τ, ψ_I, β, w and Λ for initial and final values of each stage

ZETA Vector of mass ratios for each powered stage

ZINV Inverse of weighting function matrix

ZKA Matrix relating changes in terminal constraints to changes in control parameters

ZKAL Vector relating changes in terminal constraints to changes in launch mass

4. PROGRAM OPERATION

DATA INPUT FORMAT

Data Blocks

Data required for PRESTO execution are grouped and input in data blocks which are identified by number. Each block contains a common type of information such as the case title or a stage thrust table, and utilizes a specified FORTRAN format for the entire data block. The contents of each data block are discussed and then summarized on the next several pages.

Header Cards

Input data are punched on cards which are placed behind the program binary deck. Cards for each data block are preceded and identified by a "header" card. The format of the header cards is 4I3, where the first field is the data block number and the second and third fields give the (inclusive) locations within the data block that the subsequent data cards are to be placed. Thus, if the user wished to change constants 2, 3 and 4 in data block 10, the header card would be punched _ 10 _ _ 2 _ _ 4 _ _ _ , and the first three fields of the next card would contain the three constants. The fourth field of the header card is used to identify the vehicle stage number for the data, where necessary. The stage designation should be included only for the data blocks as specified on the following pages.

Card Sequencing

Cards within each data block must follow sequentially, but data blocks may be arranged in any order. A blank card must follow the last data card.

Successive Cases

Successive cases can be run on PRESTO with a minimum of additional data input. Only the changes in data from the preceding case must be input. A blank card must follow the data for each case.

999 Card

A card with 999 punched in the first three columns must end the data deck. It is to be placed behind the blank card which ends the data for the final case. When the READ routine encounters the 999 card, a normal stop is indicated to the machine operator, so that the job can be terminated.

The following discussion will emphasize only the changes in the input with respect to the PRESTO input. The summary definition of the data blocks will be complete.

Data Block 4

(4) For planar missions, optimize theta only. For non-planar missions, optimize both theta and chi.

(11) The $\bar{q}$-alpha constraint is no longer available. If the first stage is being used, the zero-alpha constraint is automatically applied during PRESTO II iterations and must, therefore, be specified in the input so that the final PRESTO runs will be correct. Positions 5 and 6 in data block 22 must correspond to the burn time for the first stage.

(18) This option indicates whether or not there is a coast stage between the two control segments.

(19) If the accuracy of a numerically integrated trajectory is not required, the program will terminate at the end of the last PRESTO II iteration.

(20) Checkout matrices and the integration of the first stage can be output with this option.

Data Block 5

Stage 5 is no longer available. Stage 1 is always the numerically integrated stage. For orbital launches, stages 2, 3 or 4 may be used as the first stage.

Data Block 6

There is only one adjustable burn in stage 4 and only one adjustable coast can be specified. Parameter codes 1 and 2 are not available.

Data Block 8

Allow no more terminal constraints (including mass) than the number of adjustable controls.

Data Block 24

The nominal value of coast time is specified in addition to the coast angle. The program will iterate to the input time on the nominal trajectory. The input angle is an estimate of the coast angle corresponding to the desired time.

Data Block 27

The recommended weighting functions are the following:

Name	Code	Weighting Function
Coast	3,5,6,7	10^{-6}
Burn	4	10^{-3}
Launch Time	8	10^{-9}
Azimuth	9	10^{-1}
Flight Path Angle	10	500.
Initial theta in first segment	11	1.
Slope of theta in first segment	12	1000.
Initial chi in first segment	13	.1
Slope of chi in first segment	14	1000.
Initial theta in second segment	15	.1
Slope of theta in second segment	16	10 to 1000
Initial chi in second segment	17	.1
Slope of chi in second segment	18	10 to 1000

Data Block 30

Thrust must be constant except for the first stage.

Data Blocks 31, 32, 33

The only aerodynamic coefficient to be used is the drag coefficient for stage 1.

Data Block 40

The first and last values of theta for each control segment are specified. If there is no coast between the two control segments, theta must be continuous, i.e., the last value of the first segment must equal the first value of the second segment.

Data Block 41

Same as 40 for chi.

Data Block 42

The recommended value for the perturbation in the average value of a control angle is .01 radians. For a slope, the recommended value is .0001 radians/sec. If a stage is quite short, a larger perturbation should be used for the slope. The following values are suggested for the other perturbations:

Coast time	5 sec
Burn time	1 sec
Azimuth	.01 rad
Flight path angle	.001 rad

Data Block 43

DELGAM is the separation between entries in the table for stage 1. Provision has been made for 25 entries on each side of the initial value. A value of .1 degree is sufficient for vehicles with an initial flight path angle ranging from 87 to 89 degrees. A larger value for DELGAM can be used if the initial flight path angle is lower.

NEWKM1 is the number of gamma table entries in between evaluation of a new sensitivity for launch weight perturbations. 3 is a satisfactory value for a DELGAM of .1.

DELPSI is the change in azimuth required before a new sensitivity to initial azimuth perturbations is computed. Ten degrees is the suggested value.

Data Block 44

This data block determines the number of control segments to be used and the stages contained in each. The first number is the KK4 corresponding to the first powered stage in segment one. This is followed by the KK4's of the remaining stages in the first segment, including coasts. Six is input at the end of the first segment. If there is only one segment, a 7 follows the 6. If there are two segments, the KK4's of the stages in the second segment follow the 6 and a 7 terminates the data block.

Example: If the stage sequence in data block 5 is

1 6 2 7 3 4 8 4

and two segments are to be used with stage 8 in between the two segments, data block 44 should contain

2 9 3 4 6 5 7

Data Block 45

An expected average altitude must be input for each stage.

DATA INPUT

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
1	Main Heading	12A6	
2	Case Heading	12A6	
3	Date	2A6	
4	Options	24I1	IP(n) - See list of available options
5	Stage Sequence	12I1	ISTGE(n) Powered flight stages 1 through 4 Closed-form coast stages 6 through 9 * End sequence with zero
6	Adjustable Parameters	11I3	IB1(n) IB1(1) = The number of adjustable parameters to be optimized IB1(2) = The adjustable parameter codes IB1(11) in non-decreasing order

** Adjustable Parameter Codes **

3 Length of coast after first burn in stage 4
4 Length of first burn in stage 4
5 Length of coast after stage 3
6 Length of coast after stage 2
7 Length of coast after stage 1
8 Launch time of day
9 Initial azimuth
10 Initial flight path angle

8	Terminal Constraints	7I3	IB3(n) - Mission-Dependent IB3(1) = Stopping parameter code IB3(2) = Form or number of terminal constraints IB3(3) = List of constraint codes for : orbit-injection and lunar IB3(7) descent missions

** Stopping Parameter Codes **

A. Orbit-Injection Mission

1 TWOE ($V_I^2 - 2\,\mu/R$)
2 Available
3 Available
4 Inertial velocity (V_I)
5 Available
6 Velocity

B. Lunar Transfer Mission

1 Transfer time to lunar radial distance

C. Lunar Descent Mission

Use only Code 6 (Velocity)

D. Planetary Transfer Mission

1 Hyperbolic Excess Velocity (VH)

** Form or Number of Terminal Constraints **

A. Orbit-Injection Mission
and
C. Lunar Descent Mission

The number of terminal constraints (1 through 6)

* Include the terminal mass constraint in this count

B. Lunar Transfer Mission

1 Use Ephemeris, do not constrain inclination of transfer orbit
2 Use Ephemeris, do constrain inclination

D. Planetary Transfer Mission

1 Do not use Ephemeris (input VH vector)
2 Mars
3 Venus

** Constraint Codes for Orbit-Injection and Lunar Descent Missions **

1 Perigee radius
2 Orbit inclination
3 Longitude of ascending node
4 Argument of perigee
5
6 Altitude
7 Inertial flight path angle
8 Longitude
9 Inertial azimuth
10 Latitude
11 Available
12 Available
13 Available
14 Radius

DATA INPUT

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
9	Output Frequency	24I3	IB4(n) Frequency defined as: $\frac{\text{(number of computed points)}}{\text{(number of output points)}}$ IB4(1) - Initial trajectory IB4(3) - Final IB4(4) - Stage code IB4(5) - Initial trajectory IB4(7) - Final IB4(8) - Stage code IB4(9) - Initial trajectory IB4(11)- Final IB4(12)- Stage code IB4(13)- Initial trajectory IB4(15)- Final IB4(16)- Stage code
10	Nominal Constants	11E12.8	CT1(n) Nominal values shown Input only if changes are desired CT1(1) = 7.29211E-5 rad/sec omega CT1(2) = 1.407735E16 ft3/sec^2 FMU CT1(3) = 20902900. ft RE CT1(4) = 1716.4827 gas constant for atmosphere subroutine CT1(5) = 32.154856 ft/sec^2 measured sea level gravity; weight-to-mass conversion factor (GO) CT1(6) = 2116.2 lb/ft^2 PO sea level ambient pressure CT1(7) = 250,000. ft HAERO Altitude above which all aero-dynamic computations are bypassed CT1(8) = 46. Upper limit of lines per page CT1(9) = (Available) CT1(10)= .02 sec TIMEP Time-epsilon increment to insure hitting critical time

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
11	Atmosphere	8E12.8	CT2(n) Nominal values tabulated
12	Subroutine	8E12.8	CT3(n) in report section
13	Constants	8E12.8	CT4(n) describing atmosphere
14	Constants	8E12.8	CT5(n) subroutine *Input only if changes from nominal values are desired
15	Nominal Constants	8E12.8	CT6(n) *Input only for change Used in forming partial derivatives across closed-form coasts CT6(1) = 1.0 Velocity increment CT6(2) = .001 Gamma increment CT6(3) = 100. Radius increment CT6(4) = .001 Tau increment CT6(5) = .1 Mass increment CT6(6) = .001 Psi increment CT6(7) = .001 Lambda increment CT6(8) = .001 Coast angle increment
16	Time Duration and Number of Integration Steps Within Each Stage	12E12.8	DA1(n) Time in seconds DA1(1) Stage 1 time duration DA1(2) Stage 1 points DA1(3) Stage 2 time DA1(4) Stage 2 points DA1(5) Stage 3 time DA1(6) Stage 3 points DA1(7) Stage 4 total time duration DA1(8) Stage 4 total points DA1(9) Stage 4 first burnout time
17	Input Data	20E12.8	DA2(n) DA2(1) Input should = GO DA2(2) = +1.0 for takeoff = -1.0 for retro-burns DA2(17) Magnitude for total time constraint
19	Terminal Constraint Magnitudes	6E12.8	DA4(n) Mission-Dependent DA4(1) is the magnitude of the stopping parameter DA4(2) are the desired values of the : terminal constraints, listed in DA4(6) the same order as in Data Block Number 8

* Units are feet, radians, seconds

DATA INPUT

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS

** Terminal Constraints Input Guide **

A. Orbit-Injection and Lunar Descent Missions
and
C. As Described Above

B. Lunar Transfer Mission

DA4(1) Transfer time (hours)
DA4(2)
DA4(3) Used internally
DA4(4) Inclination of transfer orbit (radians)

D. Planetary Transfer Mission

DA4(1) Hyperbolic excess velocity
DA4(2) Right ascension of hyperbolic asymptote (radians)
DA4(3) Declination of hyperbolic asymptote (radians)
DA4(6) Transfer time (days) (DA4(4) and DA4(5) are used internally)

* DA4(1,2,3) are not required input if PLANEP routine is called

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
20	Initial Times	2E12.8	DA5(n) DA5(1) Time duration for closed-form lift-off calculation (sec) DA5(2) Launch time in Space Age Date (days from January 0.0, 1960)
21	Initial Conditions	6E12.8	DA6(n) DA6(1) Altitude (feet) DA6(2) Longitude (degrees ± from prime meridian) DA6(3) Latitude (degrees) DA6(4) Velocity (feet/second) DA6(5) Azimuth (degrees) DA6(6) Flight path angle (degrees)
22	Control Variable Constraints	6E12.8	DA7(n) For α = 0 constraint DA7(5) Time to start testing DA7(6) Time to release constraint
23	Constraints on State Variables During Boost	4E12.8	DA8(n) DA8(1) Available DA8(2) Available DA8(3) Minimum altitude for coast stages 8 and 9 (feet) DA8(4) Available

DATA INPUT

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
24	Nominal Range Angles and Time for Closed-Form Coast Stages	8E12.8	DA9(n),n=1,4 Angles in radians of central arc DA9(1) Coast stage 6 DA9(2) Coast stage 7 DA9(3) Coast stage 8 DA9(4) Coast stage 9 DA9(n)n=5,8 Coast time in seconds DA9(5) Coast stage 6 DA9(6) Coast stage 7 DA9(7) Coast stage 8 DA9(8) Coast stage 9
27	Weighting Constants for Control Variables	19E12.8	DA12(n) DA12(3) : DA12(10) For adjustable parameters identified by code number as listed for Data Block 6 DA12(11) Theta in segment 1 DA12(12) Theta slope in segment 1 DA12(13) Chi in segment 1 DA12(14) Chi slope in segment 1 DA12(15) Theta in segment 2 DA12(16) Theta in segment 2 DA12(17) Chi in segment 2 DA12(18) Chi slope in segment 2
28	Convergence Data	6E12.8	DA13(n) DA13(1) Initial weight improvement to be used if (automatic) internal computation fails DA13(2) Epsilon-weight for stopping attempted mass improvement DA13(3) Maximum number of forward trajectories per case DA13(4) Velocity below which Runge Kutta integration is always used DA13(5) Velocity to start open-loop computations
29	Permitted Values	5E12.8	DA14(1) : DA14(5) List in same order as constraints are specified and in the same units * Suggest 10,000 feet for distances and one degree on angles
30	Vacuum Thrust Tables	12E12.8	TT1, TT2, TT3, TT4 * Stage code required on header cards Sequence: Blank, time, thrust, time, thrust..........1.0E10

DATA INPUT

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
31	Drag Coefficient Tables (ALPHA = 0)	22E12.8	TD1 * Stage code required on header cards Sequence: Blank, Mach, C_D, Mach, C_D, Mach.......1.0E10
39	Stage Data	5E12.8	SG1, SG2, SG3, SG4, each dimensioned 5 * Stage code required on header cards SGx(1) Vacuum I_{sp} (sec) SGx(2) Aerodynamic reference area (ft^2) SGx(3) Total nozzle exit area (ft^2) SGx(4) Initial weight (lb) SGx(5) Final weight (lb)
40	Theta History	4E12.8	* CNTRL1(n) Units of degrees CNTRL1(1) Initial value for first segment CNTRL1(2) Final value for first segment CNTRL1(3) Initial value for second segment CNTRL1(4) Final value for second segment
41	Chi History	4E12.8	* CNTRL2(n) Units of degrees CNTRL2(1) Initial value for first segment CNTRL2(2) Final value for first segment CNTRL2(3) Initial value for second segment CNTRL2(4) Final value for second segment *When making a change in theta or chi, the entire table must be input.
42	Perturbations Also used in mass improvement calculation	8E12.8	DELTA(n) DELTA(1) Average value of theta and chi in first segment DELTA(2) Slope of theta and chi in first segment DELTA(3) Average value of theta and chi in second segment DELTA(4) Slope of theta and chi in second segment DELTA(5) Flight path angle DELTA(6) Azimuth DELTA(7) Coast DELTA(8) Fourth stage burn
43	Stage 1 Controls	3E12.8	TABLEG(n) TABLEG(1) = DELGAM in degrees TABLEG(2) = NEWKM1 TABLEG(3) = DELPSI in degrees

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
44	Stage Sequence in Control Segments	10I3	ISEG(n) KK4 of stages in control segments. Place 6 in between segments and 7 at end.

Stage Number	KK4
2	2
3	3
4 - 1st burn	4
4 - 2nd burn	5
6	8
7	9
8	10
9	11

DATA BLOCK NUMBER	TITLE	FORMAT	COMMENTS
45	Initial Average Altitude for Powered Stages	5E12.8	HAVI(n) HAVI(2) Stage 2 HAVI(3) Stage 3 HAVI(4) Stage 4 - first burn HAVI(5) Stage 4 - second burn

DATA INPUT

** LIST OF AVAILABLE OPTIONS FOR DATA BLOCK NUMBER 4 **

IP(1)	Mission	1	Orbit-injection
		2	Inject into lunar transfer from Earth liftoff
		3	Inject into lunar transfer from Earth orbit
		4	Lunar landing
		5	Inject into planetary transfer from Earth liftoff
		6	Inject into planetary transfer from Earth orbit
IP(2)	Axes	0	Rotating and 3D
		2	Non-rotating and 3D
IP(3)		0	
IP(4)	Control Variables to be optimized	0	Theta only
		1	Theta and Chi
IP(5)	Closed-Form Liftoff Computation	0	Use this computation
		1	Do not use this computation
IP(6)	Thrust	0	Table look-up - vacuum thrust vs time
IP(7)	Mass Computation	0	MDOT = $T_V/(G_0 * I_{sp})$
IP(8)	Aerodynamics	0	Lift = 0. only
IP(9)	Frequency of New Adjoint Solution	1	After every successful forward trajectory
IP(10)	Special Computations and Printout	0	None
IP(11)	Constraints on Control Variables	0	None
		2	ALPHA = 0.0
IP(12)	Constraints on State Variables	0	None
IP(13)	Total Time Constraint	0	Do not constrain total time
		1	Do constrain total time
IP(14)	Mass-Improvement Procedure	0	Fixed launch weight
		1	Fixed final weight

** LIST OF AVAILABLE OPTIONS **

IP(15)	Memory Dump	0	Do not dump memory
		1	Dump memory at end of job (includes floating-point dump of floating-point numbers in common)
IP(16)	Output	1	Non-buffered output
IP(17)	Output of Input Data	0	Do not output data
		1	Output data
IP(18)	Coast Between Control Segments	0	No coast between segments
		1	Coast between segments
IP(19)	Make Final Iterations on PRESTO	0	Do not call PRESTO
		1	Call PRESTO
IP(20)	Output of Check Matrices	0	Do not output check matrices
		1	Output check matrices

Changes in Program Use as Compared to PRESTO

There are several changes and limitations in the use of PRESTO II as compared to PRESTO. These are listed here.

1. A cylinder is not a good representation for the Earth if large lateral maneuvers are made. The accuracy of PRESTO II will, therefore, fall off as the size of the lateral maneuver increases.

2. The approximation concerning the $\frac{uv}{r}$ term in the equations of motion is not good when one stage goes from orbital speed to a high supercircular velocity. The program should not be used for interplanetary transfers requiring a large hyperbolic excess velocity. Program operation is satisfactory for lunar transfer missions.

3. When using the lunar landing mission option, the horizontal component of velocity must not be allowed to go negative. This can happen if a poor nominal is used or if the stopping parameter is too close to zero. Interpolation to the correct stopping condition fails when the velocity goes negative.

4. When the total time constraint option is used, the weighting function associated with the final coast stage must be set to a large number, e.g., 10^8.

5. DERIVATION AND PROGRAMMING OF OPTIMIZATION EQUATIONS

The optimization procedure used in PRESTO II is similar to that used in PRESTO. The major differences are:

1. The thrust orientation angles are now discrete rather than continuous; therefore no integral terms appear.
2. The calculation for new controls is made only once at the beginning of a trajectory. There is no closed-loop calculation as in PRESTO.
3. The sensitivity of terminal constraints to a change in a control is determined by perturbing the control and then comparing the perturbed terminal constraint with the nominal value. This process replaces the backward integration runs in PRESTO.

The fundamental relation is

$$d\Psi = K d\epsilon + L\, dm_i \tag{1}$$

where

$d\Psi$ is the terminal constraint vector

$d\epsilon$ is the control vector

dm_i is the perturbation in launch mass

K is a matrix of sensitivities, or influence coefficients

L is a vector of influence coefficients for launch mass perturbations

A trajectory terminates when a stopping parameter, S, is satisfied. The influence of the controls and the launch mass on the stopping parameter is represented by the equation

$$dS = M d\epsilon + N dm_i \tag{2}$$

It is not efficient to interpolate to the correct value of the stopping parameter when perturbing the controls. The reason for this is that each attempt at interpolation requires the same computation that is made to integrate the motion for the entire stage. Therefore, the sensitivity matrices will be evaluated for a fixed value of the mass ratio, ρ . They will then be converted to sensitivities with respect to a fixed stopping parameter. The subscripts ρ and S will be used to indicate whether the matrix refers to a fixed mass ratio or fixed stopping parameter condition.

The following equations relate terminal perturbations at a fixed stopping parameter to the perturbations at a fixed mass ratio.

$$d\Psi\big|_S = d\Psi\big|_\rho + \Psi'\big|_\rho \, d\rho \tag{3}$$

$$dS\big|_S = dS\big|_\rho + S'\big|_\rho \, d\rho \tag{4}$$

where $\Psi' = \dfrac{d\Psi}{d\rho}$; $S' = \dfrac{dS}{d\rho}$

and $d\rho$ is the change in the nominal mass ratio required to meet the stopping condition in the presence of a control perturbation.

By definition, $dS\big|_S = 0$. Solving Eq. 4 for $d\rho$, one obtains

$$d\rho = -\frac{dS\big|_\rho}{S'} \tag{5}$$

Substitute Eq. 5 into Eq. 3 to obtain

$$d\Psi\Big|_{S} - d\Psi\Big|_{\rho} - \frac{\Psi'}{\rho'}\, dS\Big|_{\rho} \tag{6}$$

Eqs. 1 and 2 are to be evaluated for fixed mass ratios. Using the proper subscripts, substitute then into Eq. 6 to obtain

$$d\Psi\Big|_{S} = K_S\, d\epsilon + L_S\, dm_i \tag{7}$$

where

$$K_S = K_\rho - \frac{\Psi'}{\rho'} M_\rho$$

$$L_S = L_\rho - \frac{\Psi'}{\rho'} N_\rho$$

It is now desired to find the vector $d\epsilon$ which will produce a desired change $d\Psi\big|_S$ in the presence of a launch mass change dm . Furthermore, a weighted sum of the squares of the changes in the control parameters should be minimized, i.e., one desires to minimize

$$d\epsilon^T Z\, d\epsilon$$

while satisfying Eq. 7. Z is a diagonal matrix of weighting functions. Use Lagrange multipliers to form the quantity

$$V = d\epsilon^T Z\, d\epsilon + \mu\left(d\Psi\Big|_S - K_S\, d\epsilon - L_S\, dm_i\right) \tag{8}$$

Write the first order perturbation equation for V

$$\delta V = \left(2\, d\epsilon^T Z - \mu K_s\right) \delta(d\epsilon) \tag{9}$$

δV must be zero for arbitrary changes in $d\epsilon$. The coefficient of $\delta(d\epsilon)$ must therefore be zero. Solving for $d\epsilon$ gives the following result.

$$d\epsilon = \frac{Z^{-1} K_s^T \mu^T}{2} \tag{10}$$

To determine μ , substitute Eq. 10 into Eq. 7.

$$\mu^T = 2\left(K_s Z^{-1} Z_s^T\right)^{-1} \left(d\Psi\big|_s - L_s\, dm_i\right) \tag{11}$$

Substitute Eq. 11 into Eq. 10 to obtain

$$d\epsilon = Z^{-1} K_s^T \left(K_s Z^{-1} K_s^T\right)^{-1} \left(d\Psi\big|_s - L_s\, dm_i\right) \tag{12}$$

In the program, the following notation has been used

$K_\rho = \phi KZ$	$K_s = \phi KS$
$L_\rho = \phi LZ$	$L_s = \phi LS$
$M_\rho = \phi MZ$	$M_s = \phi MS$
$N_\rho = \phi NZ$	$N_s = \phi NS$
$\Psi' = PSIPR$	$S' = SPR$
$\Psi'/S' = PDS$	$K_s Z^{-1} Z_s^T = AA$
$Z^{-1} K_s^T (K_s Z^{-1} K_s^T)^{-1} = ZKA$	$(K_s Z^{-1} Z_s^T)^{-1} = AA$
$Z^{-1} K_s^T (K_s Z^{-1} K_s^T)^{-1} L_s = ZKAL$	$Z^{-1} = ZINV$

K_ρ, L_ρ, M_ρ, N_ρ, Ψ', and S' are evaluated in the PERTRB subroutine. K_ρ and M_ρ are determined by perturbing the controls, one by one, and observing the effect on the terminal constraints.

L_ρ and N_ρ are determined in the same way with the launch mass perturbed. Ψ' and S' are evaluated by perturbing the mass ratio of the final stage.

K_S, L_S, M_S and N_S are computed in the MEQ2 subroutine. AA is formed and inverted and substituted back into AA. ZKA and ZKAL are then formed.

ZINV is computed at the start of the main program. The actual computation of the change in the control vector (Eq. 12) is made in the CØNC subroutine in the form

$$DC\O N = ZKA\, d\Psi - ZKAL\, dm_i \tag{13}$$

where DCØN is the program symbol for $d\epsilon$.

OPTIMIZATION SUCCESS CRITERION

The optimization success criterion is identical to that used in PRESTO. Only the notation has been changed. Thus, Eq. 1 on page 8-14a in the PRESTO report becomes

$$dm_f = PPAY_1\, dm_i + \sum_{j=2}^{JC} PPAY_j\, d\Psi_j \tag{14}$$

An optimization run is successful if the terminal constraints are sufficiently close to the desired values and if

$$(m_f + dm_f)_{\text{CURRENT RUN}} > (m_f + dm_f)_{\text{LAST RUN}} \tag{15}$$

where $dm_f = \sum_{j=2}^{JC} PPAY_j \, d\Psi_j$ for the fixed launch weight option

and $dm_f = \left\{ \sum_{j=2}^{JC} PPAY_j \, d\Psi_j + PPAY_1 (DWFUEL/g_o) \right\} (1 - PPAY_1)^{-1}$

for the fixed final stage option.

SELECTION OF MASS IMPROVEMENT

The selection of mass improvement differs from PRESTO in that the improvement to be asked for is recomputed after every successful run using the relation

$$dm_{NOM} = \sqrt{\frac{dP^2}{A_{11}}} \tag{16}$$

dm_{NOM} is then added to dm_f to obtain $d\Psi_1$, which is the actual change in final mass requested. If a run is unsuccessful, dm_{NOM} is cut in half. In the output, DMASD still indicates the first computation of dm_{NOM} made after a successful guidance run.

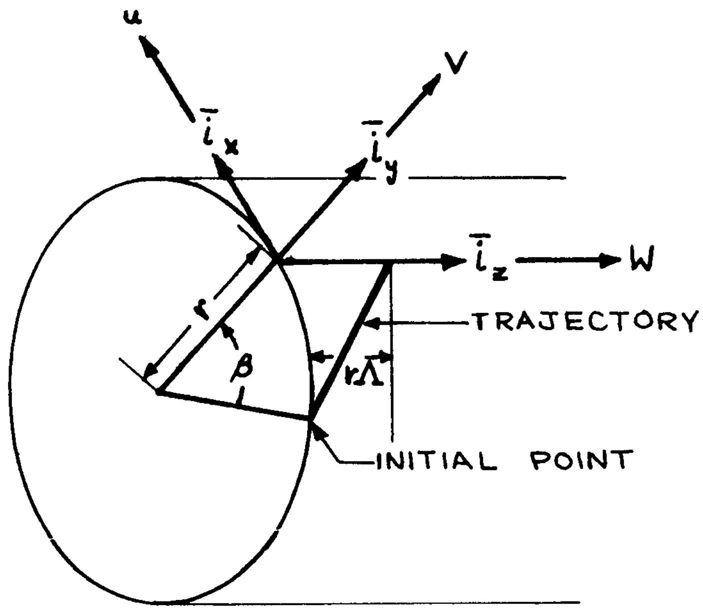
6. EQUATIONS OF MOTION

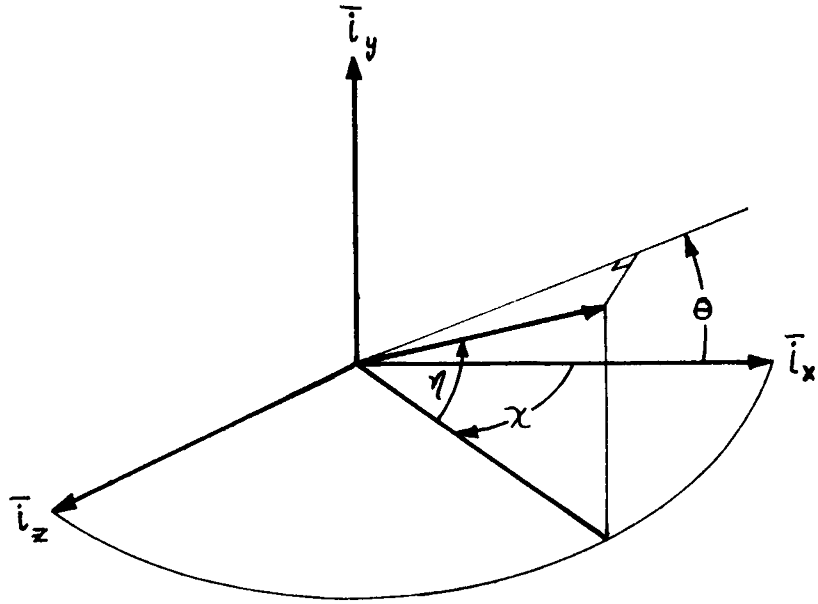
AN APPROXIMATE SOLUTION TO THE THREE-DIMENSIONAL EQUATIONS OF MOTION OUTSIDE OF THE ATMOSPHERE

If motion is restricted to the vicinity of a great circle defined by the initial velocity vector, it is reasonable to represent the Earth as a cylinder. On a cylindrical Earth, motion in the longitudinal and lateral directions is uncoupled. Longitudinal motion may be described in the usual polar coordinate system while the lateral motion takes place under the assumption that the Earth is flat.

The coordinate system used in PRESTO II is shown on the next page. The initial velocity vector lies in the $\bar{i}_x$ - $\bar{i}_y$ plane. The $\bar{i}_x$ axis is along the local horizontal and the $\bar{i}_y$ axis lies along the radius vector. u, v, and w are the components of velocity along the $\bar{i}_x$, $\bar{i}_y$, and $\bar{i}_z$ axes, respectively. β is the in-plane range angle. Λ is the out-of-plane distance divided by the radius.

The thrust orientation angles are illustrated on the next page. θ is the angle between the $\bar{i}_x$ axis and the component of thrust in the $\bar{i}_x$ - $\bar{i}_y$ plane. χ is the angle between the $\bar{i}_x$ axis and the component of thrust in the $\bar{i}_x$ - $\bar{i}_z$ plane. η is the angle between the thrust vector and the local horizontal. It is related to the two control angles by the Equation:





DIAGRAMS OF COORDINATE SYSTEM AND THRUST ORIENTATION ANGLES

$$\tan \eta = \tan \theta \cos \chi \tag{1}$$

With lift and drag neglected, the equations of motion are:

$$\dot{u} = \frac{T}{m} \cos \eta \cos \chi \operatorname{sgn} \xi - \frac{uv}{r} \tag{2}$$

$$\dot{v} = \frac{T}{m} \sin \eta \operatorname{sgn} \xi - g + \frac{u^2}{r} \tag{3}$$

$$\dot{w} = \frac{T}{m} \cos \eta \sin \chi \operatorname{sgn} \xi \tag{4}$$

$$\dot{\beta} = \frac{u}{r} \tag{5}$$

$$\dot{r} = v \tag{6}$$

$$\dot{\Lambda} = \frac{w}{r} \tag{7}$$

$$\dot{m} = -\frac{T}{c} \tag{8}$$

Sgn ξ is normally +1. It is set to -1 for retroburns because θ and χ are limited to ±90 degrees.

It will be convenient to use the mass ratio, instead of time, as the independent variable. The mass ratio is defined as:

$$\mathfrak{f} = \frac{m}{m_i} \tag{9}$$

where m_i is the mass at the beginning of a stage. The time derivative of $\mathfrak{f}$ is:

$$\frac{d\rho}{dt} = -\frac{T}{m_i c} \qquad (10)$$

With the help of Eq. 10, the equations of motion are converted to:

$$\frac{du}{d\rho} = -\frac{c \cos\eta \cos\chi \operatorname{sgn}\xi}{\rho} + \frac{m_i c}{T r} uv \qquad (11)$$

$$\frac{dv}{d\rho} = -\frac{c \sin\eta \operatorname{sgn}\xi}{\rho} - \frac{m_i c}{T} \frac{u^2}{r} + \frac{g m_i c}{T} \qquad (12)$$

$$\frac{dw}{d\rho} = -\frac{c \cos\eta \sin\chi \operatorname{sgn}\xi}{\rho} \qquad (13)$$

$$\frac{d\beta}{d\rho} = -\frac{m_i c}{T} \frac{u}{r} \qquad (14)$$

$$\frac{dr}{d\rho} = -\frac{m_i c}{T} v \qquad (15)$$

$$\frac{d\Lambda}{d\rho} = -\frac{m_i c}{T} \frac{w}{r} \qquad (16)$$

$$\frac{dm}{d\rho} = m_i \qquad (17)$$

where c is the exhaust velocity.

The two thrust orientation angles, θ and χ, are to vary in accordance with the "linear-tangent" law; i.e.,

$$\tan\theta = a + bt \qquad (18)$$

$$\tan \chi = e + ft \tag{19}$$

a, b, e, and f are constants.

Using the relation

$$m = m_i - \frac{T}{c} t \tag{20}$$

one can write

$$\tan \theta = A + B\xi \tag{21}$$

$$\tan \chi = E + F\xi \tag{22}$$

where $A = a + \frac{bcm_i}{T}$ $\qquad$ $E = e + \frac{fcm_i}{T}$

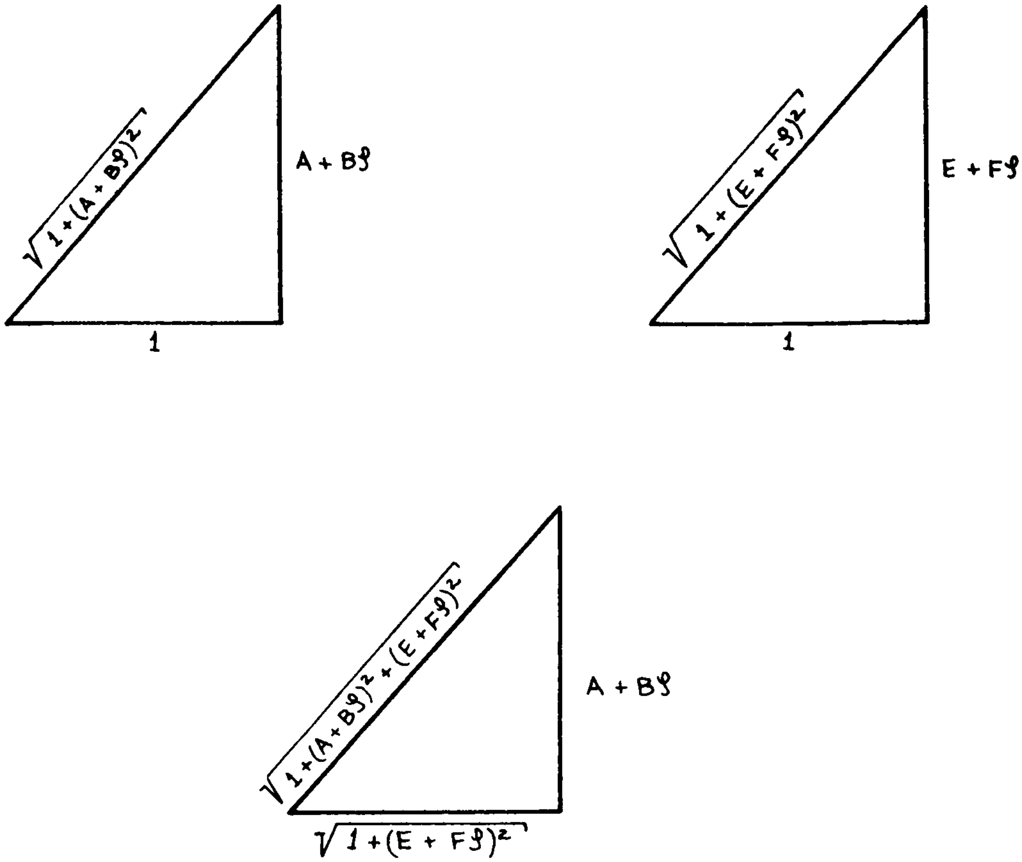
$B = - \frac{bcm_i}{T}$ $\qquad$ $F = - \frac{fcm_i}{T}$

With the aid of the following three triangles, the trigonometric terms in the equation of motion can be written as:

$$\cos \eta \cos \chi = \frac{1}{\sqrt{1 + (A + B\xi)^2 + (E + F\xi)^2}} \tag{23}$$

$$\sin \eta = \frac{A + B\xi}{\sqrt{1 + (A + B\xi)^2 + (E + F\xi)^2}} \tag{24}$$

$$\cos \eta \sin \chi = \frac{E + F\xi}{\sqrt{1 + (A + B\xi)^2 + (E + F\xi)^2}} \tag{25}$$



The equation for the horizontal component of velocity (Eq. 11) is first integrated with the term containing uV neglected. One has,

$$u_f - u_i = -c \,\mathrm{sgn}\,\xi \int_1^{\zeta_f} \frac{d\zeta}{\zeta\sqrt{1+(A+B\zeta)^2+(E+F\zeta)^2}}$$

After integration, one obtains

$$u_f - u_i = \frac{c\,\mathrm{sgn}\,\xi}{\sqrt{1+A^2+E^2}} \times \tag{26}$$

where

$$X = \ln\left[\frac{\sqrt{1+(A+B\varsigma)^2+(E+F\varsigma)^2}\,\sqrt{1+A^2+E^2}+1+A(A+B\varsigma_f)+E(E+F\varsigma_f)}{\sqrt{1+(A+B)^2+(E+F)^2}\,\sqrt{1+A^2+E^2}+1+A(A+B)+E(E+F)}\;\frac{1}{\varsigma_f}\right]$$

For β , one has

$$\beta_f - \beta_i = -\frac{m_i c}{T r}\int_1^{\varsigma_f} u\,d\varsigma = -\frac{m_i c}{T r}\int_1^{\varsigma_f}\left[u_i + \int_1^{\varsigma_f}\frac{du}{d\varsigma}\,d\varsigma\right]d\varsigma$$

$$= -\frac{m_i c}{T r}\,u_i(\varsigma_f - 1) - \frac{m_i c}{T r}\int_1^{\varsigma_f}\int_1^{\varsigma_f}\frac{du}{d\varsigma}\,d\varsigma\,d\varsigma$$

The double integral is evaluated with the help of the relation

$$\int_1^{\varsigma_f}\int_1^{\varsigma_f} f(\varsigma)\,d\varsigma = \int_1^{\varsigma_f}(\varsigma_f - \varsigma)\,f(\varsigma)\,d\varsigma \qquad (27)$$

Using Eq. 27, one obtains:

$$\beta_f - \beta_i = -\frac{m_i c}{T r}\left[\varsigma_f\,(u_f - u_i) + u_i(\varsigma_f - 1) + \frac{c\,\mathrm{sgn}\,\xi}{\sqrt{B^2+F^2}}\,Y\right] \qquad (28)$$

where

$$Y = \ln \left[\frac{\sqrt{1+(A+B\zeta_f)^2+(E+F\zeta_f)^2} + \sqrt{B^2+F^2}\,\zeta_f + \dfrac{AB+EF}{\sqrt{B^2+F^2}}}{\sqrt{1+(A+B)^2+(E+F)^2} + \sqrt{B^2+F^2} + \dfrac{AB+EF}{\sqrt{B^2+F^2}}} \right]$$

If B and F are both zero, the last term in Eq. 28 is:

$$\frac{c(\zeta_f - 1)\,\mathrm{sgn}\,\xi}{\sqrt{1+A^2+E^2}}$$

In order to integrate the equation for vertical velocity, an integrable expression for u^2 must be found. The approach used here is to split the logarithmic term for u into two parts and then to expand the second term in a series in ζ . One can write

$$u = u_i + \frac{c\,\mathrm{sgn}\,\xi}{\sqrt{1+A^2+E^2}} \left[\ln\frac{1}{\zeta} + \ln\left(\frac{1+A(A+B\zeta)+E(E+F\zeta)+\sqrt{1+A^2+E^2}\sqrt{1+(A+B\zeta)^2+(E+F\zeta)^2}}{1+A(A+B)+E(E+F)+\sqrt{1+A^2+E^2}\sqrt{1+(A+B)^2+(E+F)^2}} \right) \right] \tag{29}$$

Note that if the slopes of the control angles are equal to zero, the second ln term is equal to zero. Furthermore, when B and F are not zero, numerical checks for reasonable control angles indicate that the term in the round brackets does not move very far from unity. Write the term in the round brackets as N/D, where N and D are numerator and denominator, respectively. Then:

$$\ln\left(\frac{N}{D}\right) = \ln\left(\frac{D+Z}{D}\right) = \ln\left(1+\frac{Z}{D}\right) \approx \frac{Z}{D}$$

assuming N/D is close to unity. Using the approximation

$$\sqrt{1+a} \approx 1+\frac{a}{2}$$

one obtains for Z the equation:

$$Z = \left(1+\sqrt{1+A^2+E^2}\right)(AB+EF)(\rho-1)+\frac{\sqrt{1+A^2+E^2}}{2}(B^2+F^2)(\rho^2-1) \tag{30}$$

u can then be written as

$$u = u_i + \frac{c \operatorname{sgn} \xi}{\sqrt{1+A^2+E^2}} \ln\left[\frac{1}{\rho}+\frac{Z}{D}\right] \tag{31}$$

After collecting terms, one has

$$u = A_1 + A_2 \ln \rho + A_3 \rho + A_4 \rho^2 \tag{32}$$

where

$$A_1 = u_i - \frac{a_1(a_2+a_3)}{a_4}$$

$$A_2 = -a_1$$

$$A_3 = \frac{a_1 a_2}{a_4}$$

$$A_4 = \frac{a_1 a_3}{a_4}$$

and

$$a_1 = \frac{c \operatorname{sgn} \xi}{\sqrt{1+A^2+E^2}}$$

$$a_2 = \left(1 + \sqrt{1+A^2+E^2}\right)(AB+EF)$$

$$a_3 = \frac{\sqrt{1+A^2+E^2}\,(B^2+F^2)}{2}$$

$$a_4 = 1 + A(A+B) + E(E+F) + \sqrt{1+A^2+E^2}\,\sqrt{1+(A+B)^2+(E+F)^2}$$

u^2 is now readily integrable. Eq. 12 becomes:

$$\frac{dv}{d\zeta} = -\frac{c(A+B\zeta)}{\zeta\sqrt{1+(A+B\zeta)^2+(E+F\zeta)^2}} + \frac{g\,m_i c}{T} + B_1 + B_2\zeta + B_3\zeta^2 + B_4\zeta^3 + B_5\zeta^4$$

$$+ B_6 \ln\zeta + B_7(\ln\zeta)^2 + B_8\,\zeta\ln\zeta + B_9\,\zeta^2\ln\zeta \qquad (33)$$

where

$B_1 = KA_1^2$	$B_4 = 2KA_3A_4$	$B_7 = KA_2^2$
$B_2 = 2KA_1A_3$	$B_5 = KA_4^2$	$B_8 = 2KA_2A_3$
$B_3 = K(A_3^2+2A_1A_4)$	$B_6 = 2KA_1A_2$	$B_9 = 2KA_2A_4$

$$k = -\frac{m_i c}{Tr}$$

Integration of Eq. 33 yields:

$$v_f - v_i = \frac{cA\,\mathrm{sgn}\,\xi}{\sqrt{1+A^2+E^2}}X - \frac{cB\,\mathrm{sgn}\,\xi}{\sqrt{B^2+F^2}}Y + \frac{g\,m_i c}{T}(\zeta_f - 1) + P \qquad (34)$$

where

$$P = B_1(\rho_f - 1) + \frac{B_2}{2}(\rho_f^2 - 1) + \frac{B_3}{3}(\rho_f^3 - 1) + \frac{B_4}{4}(\rho_f^4 - 1) + \frac{B_5}{5}(\rho_f^5 - 1)$$
$$+ B_6\left[\rho_f(\ln \rho_f - 1) + 1\right] + B_7\left[\rho_f(\ln \rho_f)^2 - 2\rho_f \ln \rho_f + 2\rho_f - 2\right]$$
$$+ B_8\left[\frac{1}{2}\rho_f^2(\ln \rho_f - \frac{1}{2}) + \frac{1}{4}\right] + B_9\left[\frac{1}{3}\rho_f^2(\ln \rho_f - \frac{1}{3}) + \frac{1}{9}\right]$$

Using Eq. 27 once again, the equation for r is integrated to give:

$$r_f - r_i = \frac{-m_i c}{T}\left[\rho_f(V_f - V_i) + V_i(\rho_f - 1) + \frac{c\,F\,(AF - BE)\,\mathrm{sgn}\,\xi}{(B^2 + F^2)^{3/2}}\,Y\right.$$
$$+ \frac{c\,B\,\mathrm{sgn}\,\xi}{B^2 + F^2}\left(\sqrt{1 + (A + B\rho_f)^2 + (E + F\rho_f)^2} - \sqrt{1 + (A + B)^2 + (E + F)^2}\right)$$
$$- \frac{B_1}{2}(\rho_f^2 - 1) - \frac{B_2}{3}(\rho_f^3 - 1) - \frac{B_3}{4}(\rho_f^4 - 1) - \frac{B_4}{5}(\rho_f^5 - 1) - \frac{B_5}{6}(\rho_f^6 - 1)$$
$$- \frac{B_6}{2}\left[\rho_f^2(\ln \rho_f - \frac{1}{2}) + \frac{1}{2}\right] - \frac{B_7}{2}\left[\rho_f^2\left\{(\ln \rho_f)^2 - \ln \rho_f + \frac{1}{2}\right\} - \frac{1}{2}\right]$$
$$- \frac{B_8}{3}\left[\rho_f^3(\ln \rho_f - \frac{1}{3}) + \frac{1}{3}\right] - \frac{B_9}{4}\left[\rho_f^4(\ln \rho_f - \frac{1}{4}) + \frac{1}{4}\right] \tag{35}$$
$$\left. - \frac{m_i c g}{2T}(\rho_f^2 - 1)\right]$$

If B and F are zero, the term containing Y becomes:

$$\frac{c\,A\,\mathrm{sgn}\,\xi\,(\rho_f - 1)}{\sqrt{1 + A^2 + E^2}}$$

In a similar manner, the equations for w and Λ are integrated to give:

$$w_f - w_i = \frac{cE \,\mathrm{sgn}\,\xi}{\sqrt{1+A^2+E^2}} X - \frac{cF \,\mathrm{sgn}\,\xi}{\sqrt{B^2+F^2}} Y \qquad (36)$$

$$\Lambda - \Lambda_i = -\frac{m_i c}{T_r}\Big[\mathcal{S}_f(w_f - w_i) + \frac{cB(BE-AF)\,\mathrm{sgn}\,\xi}{(B^2+F^2)^{3/2}} Y \qquad (37)$$
$$+ \frac{cF\,\mathrm{sgn}\,\xi}{B^2+F^2}\left(\sqrt{1+(A+B\mathcal{S}_f)^2+(E+F\mathcal{S}_f)^2} - \sqrt{1+(A+B)^2+(E+F)^2}\right) + w_i(\mathcal{S}_f - 1)\Big]$$

If B and F are zero, the term containing Y becomes:

$$\frac{cE\,\mathrm{sgn}\,\xi\,(\mathcal{S}_f - 1)}{\sqrt{1+A^2+E^2}}$$

It will be recalled that the term containing uv in the differential equation for horizontal velocity was neglected. The influence of this term is accounted for in an approximate manner by adding to u_f the average value of $\frac{m_i c}{T_r} uv$ multiplied by the interval of integration. This additional term is:

$$\frac{m_i c}{2T_r}(u_i v_i + u_f v_f)(\mathcal{S}_f - 1)$$

This approximate solution to the equations of motion is repeated for each powered stage with the final conditions at the end of the first stage becoming the initial conditions for the second stage. If a coast precedes a powered stage, the axis of the cylinder is rotated so that there is no lateral motion at the beginning of the powered state; i.e., Λ_i and w_i are set to zero.

The value of r that is used in the solutions is obtained by averaging the initial and final radius for a stage on the last iteration. g is computed using this average value of r .

CONVERSION FROM CYLINDRICAL TO GEOGRAPHIC COORDINATES

At the end of each powered stage, it is necessary to determine the position and velocity of the vehicle with respect to the spherical Earth. This computation is done in the CONVRT subroutine.

At the beginning of the first control segment and after each coast we have available the inertial azimuth, Ψ_I , the latitude, λ , and the longitude, τ . At the end of each powered stage, we know u, v, w, β, and Λ. It is necessary to compute inertial velocity, V_I , inertial flight path angle, γ_I , inertial azimuth, latitude and longitude.

Except for the symbols mentioned above, the terminology used in this subroutine is not related to the rest of the program.

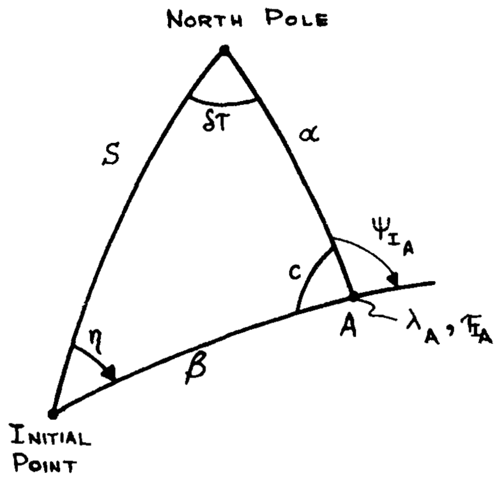
Inertial velocity is easily found from the Equation:

$$V_I = \sqrt{u^2 + v^2 + w^2} \tag{1}$$

Inertial flight path angle is found from:

$$\gamma_I = \tan^{-1} \frac{V}{\sqrt{u^2 + w^2}} \tag{2}$$

Spherical trigonometry is required to determine the remaining variables. Consider the following triangle:



The computation is broken into two parts. We first determine the latitude and longitude at point A which is on the great circle defined by the initial velocity vector. η takes on one of two values depending on Ψ_I at the initial point:

If $\Psi_I < \pi$, $\eta = \Psi_I$

If $\Psi_I > \pi$, $\eta = 2\pi - \Psi_I$

$$S = \frac{\pi}{2} - \lambda$$

Using spherical trigonometry, one obtains:

$$\cos \alpha = \cos S \cos \beta + \sin S \sin \beta \cos \eta \tag{1}$$

$$\frac{\sin \alpha}{\sin \eta} = \frac{\sin \beta}{\sin \delta\tau} \qquad (2)$$

Compute $\sin \alpha$ from (1) as:

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} \qquad (3)$$

Use Eqs. (1) and (3) to compute:

$$\alpha = \tan^{-1}\left(\frac{\sin \alpha}{\cos \alpha}\right) \qquad (4)$$

If $\tan \alpha > 0$, $0 < \alpha < \frac{\pi}{2}$

If $\tan \alpha < 0$, $\frac{\pi}{2} < \alpha < \pi$

Knowing α, solve for $\sin \delta\tau$

$$\sin \delta\tau = \frac{\sin \eta \sin \beta}{\sin \alpha} \qquad (5)$$

Then, $\cos \delta\tau = \sqrt{1 - \sin^2 \delta\tau}$

and $\delta\tau = \tan^{-1}\left(\frac{\sin \delta\tau}{\cos \delta\tau}\right)$ (6)

$\delta\tau$ is always assumed to be less than $\pi/2$.

The latitude at point A, λ_A, is:

$$\lambda_A = \frac{\pi}{2} - \alpha \tag{7}$$

and the longitude is:

$$\tau_A = \tau_I \pm \delta\tau \qquad \begin{array}{l} + \text{ if } \psi_I < \pi \\ - \text{ if } \psi_I > \pi \end{array}$$

Knowing $\delta\tau$, one can solve for c using

$$\cos c = -\cos\delta\tau\cos\eta + \sin\delta\tau\sin\eta\cos S \tag{8}$$

$$\sin c = \sqrt{1-\cos^2 c}$$

and

$$c = \tan^{-1}\left(\frac{\sin c}{\cos c}\right) \tag{9}$$

If $\tan c > 0 \qquad 0 < c < \frac{\pi}{2}$

If $\tan c < 0 \qquad \frac{\pi}{2} < c < \pi$

Then

$$\psi_{I_A} = \pi - c \tag{10}$$

The inertial azimuth at the end of a stage also depends upon the out-of-plane velocity, w. Let

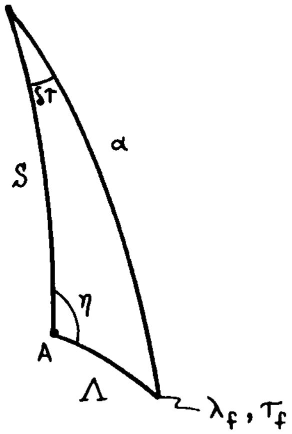
$$d = \tan^{-1}\left(\frac{W}{c}\right) \tag{11}$$

The inertial azimuth is then found from:

$$\psi_f = \psi_{I_A} + d \qquad \text{If } \psi_I < \pi$$

$$\psi_f = 2\pi - \psi_{I_A} + d \qquad \text{If } \psi_I > \pi \tag{12}$$

The second part of the calculation is similar to the first except that Λ replaces β .



Define an angle m as follows:

$$m = \psi_{I_A} + \frac{\pi}{2}\, sgn\, d \qquad \text{if } \psi_I < \pi$$

$$m = 2\pi - \psi_{I_A} + \frac{\pi}{2}\, sgn\, d \qquad \text{if } \psi_I > \pi$$

If $m > 2\pi$, subtract 2π from it.

Define η as follows:

$$\eta = m \qquad \text{if } m < \pi$$

$$\eta = 2\pi - m \qquad \text{if } m > \pi$$

$$S = \frac{\pi}{2} - \lambda_A$$

and

$$\cos\alpha = \cos S \cos\Lambda + \sin S \sin\Lambda \cos\eta \tag{13}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

$$\alpha = \tan^{-1}\left(\frac{\sin\alpha}{\cos\alpha}\right)$$

Use the quadrant check that follows Eq. 4:

Then

$$\sin \delta T = \frac{\sin\eta \sin\Lambda}{\sin\alpha} \tag{14}$$

$$\cos \delta T = \sqrt{1 - \sin^2 \delta T}$$

and

$$\delta T = \tan^{-1}\left(\frac{\sin \delta T}{\cos \delta T}\right) \tag{15}$$

The latitude at the end of the stage is:

$$\lambda_f = \frac{\pi}{2} - \alpha \tag{16}$$

and the longitude is:

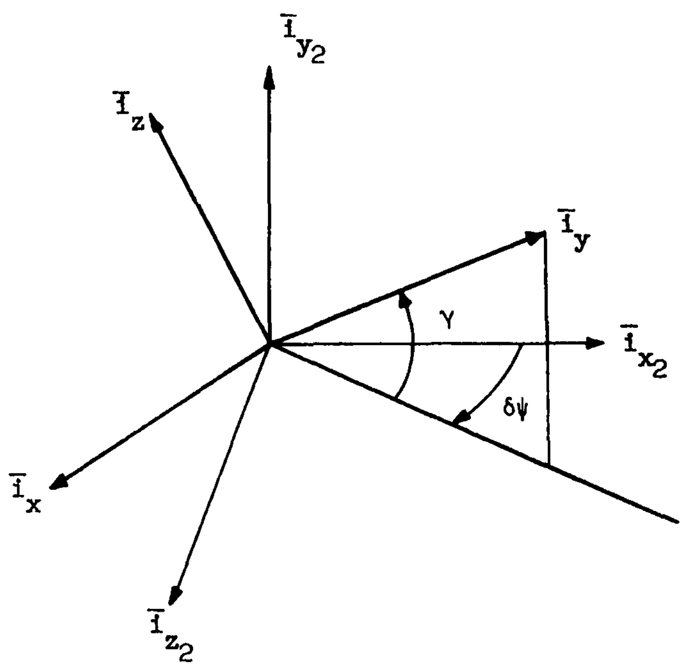
$$T_f = T_{I_A} - \omega\Delta t \pm \delta T \qquad \begin{array}{l} + \text{ if } m < \pi \\ - \text{ if } m > \pi \end{array}$$

where Δt is the time from the initial point.

CONVERSION OF PRESTO II CONTROL ANGLES TO PRESTO CONTROL ANGLES

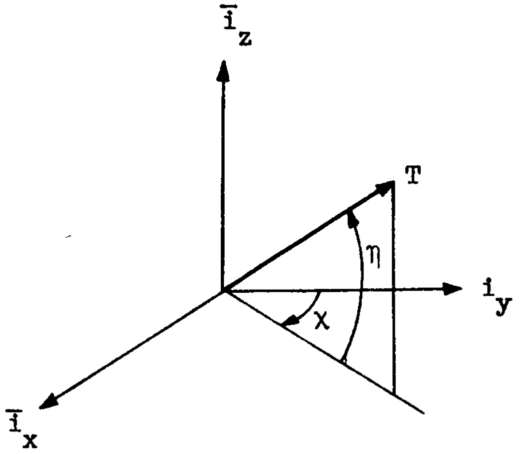
At the end of the PRESTO II iteration, we have two control angles, θ_2 and χ_2, defined with respect to the PRESTO II coordinate system. It is necessary to convert these angles to the PRESTO control angles, η and χ, for use in generating the initial integrated trajectory.

The two coordinate systems are related as shown in the following diagram. The subscript 2 refers to the PRESTO II system. γ is the flight path angle.

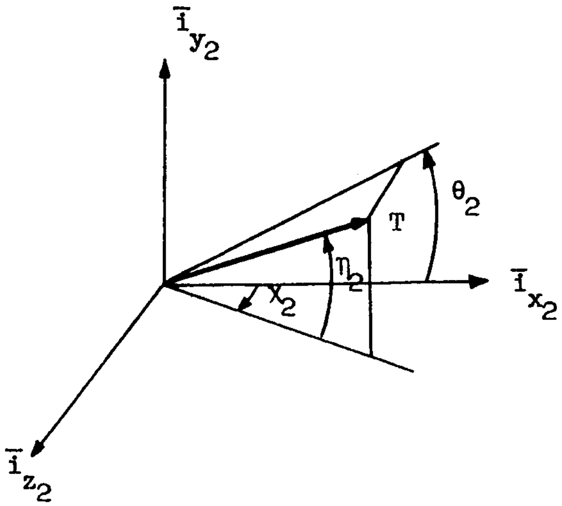


$\delta\psi$ is the angle between the great circle path defined by the initial velocity vector in the PRESTO II system and the horizontal component of velocity in the PRESTO system. The lateral motion in the PRESTO II system, described by the angle Λ is assumed to be small. This leads to the approximation that the $\bar{i}_z$ and $\bar{i}_{y_2}$ axes are parallel when γ is zero.

The orientation of the thrust vector in the two coordinate systems is shown below.



PRESTO



PRESTO II

The thrust vector in the PRESTO II system is resolved into components to give

$$\bar{T} = T\left[\bar{i}_{x_2} T_{x_2} + \bar{i}_{y_2} T_{y_2} + \bar{i}_{z_2} T_{z_2}\right] \qquad (1)$$

where $T_{x_2} = \cos \eta_2 \cos \chi_2$

$T_{y_2} = \sin \eta_2$

$T_{z_2} = \cos \eta_2 \sin \chi_2$

and $\tan \eta_2 = \tan \theta_2 \cos \chi_2$

The components of thrust in the PRESTO II system, resolved along the PRESTO coordinates, is given by

$$\bar{T} = T\left\{\bar{i}_x\left[-T_{x_2}\sin\delta\psi + T_{z_2}\cos\delta\psi\right] + \bar{i}_y\left[T_{x_2}\cos\delta\psi\cos\gamma + T_{y_2}\sin\gamma\right.\right.$$

$$\left. + T_{z_2}\sin\delta\psi\cos\gamma\right] + \bar{i}_z\left[-T_{x_2}\cos\delta\psi\sin\gamma + T_{y_2}\cos\gamma\right.$$

$$\left.\left. - T_{z_2}\sin\delta\psi\sin\gamma\right]\right\} \tag{2}$$

The components of thrust in the PRESTO system, resolved along the PRESTO coordinates, is given by

$$\bar{T} = T\left[\bar{i}_x\cos\eta\sin\chi + \bar{i}_y\cos\eta\cos\chi + \bar{i}_z\sin\eta\right] \tag{3}$$

The coefficients of the unit vectors in Eqs. 2 and 3 must be equal. One obtains

$$\sin\eta = -T_{x_2}\cos\delta\psi\sin\gamma + T_{y_2}\cos\gamma - T_{z_2}\sin\delta\psi\sin\gamma \tag{4}$$

$$\cos\eta = \sqrt{1 - \sin^2\eta} \tag{5}$$

$$\eta = \tan^{-1}\left(\frac{\sin\eta}{\cos\eta}\right) \tag{6}$$

and

$$\sin\chi = \frac{1}{\cos\eta}\left[-T_{x_2}\sin\delta\psi + T_{z_2}\cos\delta\psi\right] \tag{7}$$

$$\cos\chi = \sqrt{1 - \sin^2\chi} \tag{8}$$

$$\chi = \tan^{-1}\left(\frac{\sin\chi}{\cos\chi}\right) \tag{9}$$

These calculations are made in the PCAL subroutine.

The angle $\delta\psi$ is computed as

$$\delta\psi = \psi - \psi_R \tag{10}$$

where ψ is the local azimuth and ψ_R is the instantaneous azimuth along the great circle defined by the initial inertial velocity vector. ψ_R is computed after every step of integration using the relation

$$\psi_{R_n} = \psi_{R_{n-1}} + \dot{\psi}_R \,(DT) \tag{11}$$

where

$$\dot{\psi}_R = \frac{V_I \cos \gamma_I \sin \psi_R \sin \lambda}{r \cos \lambda}$$

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